Portfolio Risk Reduction and Skewness Effects

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ABSTRACT

Numerous studies have suggested that more investors nowadays are incorporating skewness as a factor in the selection of equity portfolios and the composition of the optimal portfolio can be significantly affected by this factor. After comprehensive literature review on the debate of this topic and the methods in trying to incorporate skewness in portfolio optimization, the paper uses empirical data first on two-asset portfolios, then on multi-asset portfolios selected based on various criteria such as industry sectors, correlation coefficient, random pick etc., to test on the effect of skewness to the risk of the portfolio. From the experiment results generated by two-asset portfolios, we find that negative skewness is the weakest in risk reduction. Then using this discovery, the paper runs regressions on the portfolio skewness to the risk reduction of the portfolio and discovered the opposite result, the two variables are actually negatively related. This means that the more positive skewed stocks are chosen in a portfolio, the smaller the portfolio risk is reduced.

INTRODUCTION

In the past few decades, a lot of papers have been published focusing on the skewness of stock returns and a lot of them have reached to the conclusion that the distributions for the returns of individual stocks are not perfectly normal but are skewed. Before this assertion, the return distribution has always been assumed to be normal and all moments higher than two are considered irrelevant to investor's decisions under uncertainty (such as Samuelson, 1970; Rubinstein, 1973; Tobin, 1958). Later empirical studies revealed that the rate of return on equities does not yield to a symmetric probability distribution (Ibbotson, 1975; Prakash et al., 2001). Also in papers by Kraus and Litzenberger, 1976; Prakash and Bear, 1986; Stephens and Proffitt, 1991, skewness, which is the third moment, has all been identified as present in the distribution of stock returns. Later on, three papers had been written in explaining the asset skewness theoretically (Grossman, Zhou 1996; Constantinides, 1997; Bates, 1996). Hence it has been proved rigorously that higher moments such as skewness on return distributions cannot be neglected. These studies point out the importance of skewness in modern finance, as the unrealistic assumption of normal distribution is wrong.

This rising debate on skewness in stock returns has also apparently drawn attention on equity portfolio selection since individual stocks are the major elements in a portfolio. H. Markowitz made a speech in

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Baruch College when he earned the Nobel Prize in 1990 for introducing the modern portfolio theory. During the speech, he admitted that if one can overcome computational difficulties, there is “more to be done” in incorporating higher moments (i.e. third and fourth moments such as skewness and kurtosis) into portfolio modeling[1]. In his modern portfolio theory, Markowitz adopts the use of mean-variance evaluation to trade off risk and return. However, a recent study by DeMiguel, Garlappi, and Uppal (2009) shows that nowadays a lot of mean-variance (MV) portfolio models designed to reduce the effect of estimation error cannot efficiently increase return or decrease risk than any other simpler portfolios. On the other hand, some like Roman, Darby-Dowman, and Mitra (2007) try to ameliorate the MV model by introducing more risk factors. Unsatisfied at all the formal approaches, some start to look at higher moments to include into the MV model. And based on the papers discussing individual stock returns, many believed that including skewness in the MV model might make a better portfolio.

Skew, or skewness can be mathematically defined as the averaged cubed deviation from the mean divided by the standard deviation cubed. The formula is expressed as the following:

$$\text{Skewness} = \frac{\sum(x_i - \bar{x})^3}{(n-1)\sigma^3}$$

If the result of the computation is greater than zero, the distribution is positively skewed. If it's less than zero, it's negatively skewed. If the result is equal to zero then it means that the distribution it's symmetric. For interpretation and analysis, skewness is always used to focus on downside risk. Negatively skewed distributions have a long left tail, which for investors can mean a greater chance of extremely negative outcomes. Positive skew would mean frequent small negative outcomes, and extremely bad scenarios are not as likely.

Study by Arditti in 1967 shows that investors prefer positive skewness rather than negative since positive skewness means a fat tail to the right and even though this means more frequent small negative returns, it greatly decreases possibilities of extreme negative returns. On behavioral finance side, stocks with a positive skewness return distributions seem more preferable especially to risk averse investors as the distribution skew to the right and extremely negative values will rarely occur. Choosing positively skewed stocks hence may significantly decrease the risk of the portfolio. However, this assumption is yet lack of empirical support and the exact effect of skewness to portfolio return and risk is still under discussion. It is still not clear if one should include as many positively skewed stocks as possible or if there should be a combination of both positive and negative in order to optimize the portfolio. The discoveries of the effect of skewed stocks in a portfolio shed lights to those who want to improve the original portfolio theory by including higher moments. It inspired the mean-variance-skewness model seekers by optimizing the skewness to an either positive of moderate level. Many methods have incorporated this optimization concept in their model.

This paper will first look at different methods used in formulating a mean-variance-skewness model on portfolios and then use empirical data to study the behavior of portfolio skewness on portfolio risk reduction.

COUNTER ARGUMENTS ON SKEWNESS PERSISTENCY OVER TIME

Many recent studies show that the market return distribution is constantly positively skewed. However, the studiy shown by J. Clay Singleton and John Wingender (1986) indicates that the skewness of individual stocks and portfolios of stocks does not persist across different time periods. Positively- skewed equity portfolios in one period are not likely to be positively skewed in the next time period. Past positively skewed returns do not predict future positively skewed returns. The most important finding in this paper is that while skewness occurs with almost constant frequency in cross sections, neither individual securities nor portfolios remain skewed over time. These results actually dispute previous evidence and are inconsistent with investment policies that encourage aggressive investors to concentrate in skewed equities.

In fact, Beedles and Simkowitz[2] (1980) have done a research on the persistency on skewness by comparing the frequency of securities with skewed returns in different cross sections. They concluded that, “regardless of how skewness is measured, securities have displayed a persistent propensity to positive asymmetry during the last three decades." However, the study by J. Clay Singleton and John Wingender shows that even thought skewness is persistent in cross section data, it is not when using time series. The paper uses time series of data from 1960 to 1980 and analyzed the skewness level of all the consistent stocks each year.

The table above indicates that skewness was less persistent for five-stock portfolios than it was for individual securities. It also noted, as did Simkowitz and Beedles, that positive skewness was much less frequent in the portfolio returns. Only 23 of the 108 twenty-stock portfolio returns were right skewed, and the skewness was almost never persistent.

This paper, assuming to be valid, gives us a warning that the attempts to select stocks based on skewness may fail, as the skewness level may not persist over time. According to this paper, in order to minimize the time series effect, we decided to focus our data on only one year (the year of 2011).

DATA SET DESCRIPTION

Before going into details, we hereby clarify the data selection of this paper and to create reference for future chapters. Our data pool is gathered from prominent research institution on the monthly return of all the US market trading stock in one year, the year of 2011. Note that we only picked the data on one year, the year of 2011. The reason is that according to the literature we studied in chapter two (section 2.3), some scholars argue that skewness does not tend to last or stay at the same level overtime. To build on this argument, this paper thus presumes that the assertions in the literature are valid and that time series data is not efficient or may be misleading in skewness study. The literature by J. Clay Singleton and John

Wingender shows that even though skewness is persistent in cross section data, it is not when using time series. Therefore, in this paper we adopt portfolios that are within a year with monthly returns.

The reason for us to pick the year 2011 specifically is because the year 2011 is the closest to today that has complete monthly data available to public. Even though studying the most recent data is always proven to be challenging, the effort may provide valuable insight towards the current market and aid current investors in their portfolio selection. This is why we did not randomly pick a year in the further past because the study result, due to uncertainties and hundreds of factors affecting the market at that year, may not have any value to portfolio investment on skewness today.

The entire pool of equities is on over 7000 stocks. We then screened off the companies that does not have continuous data or have incomplete information of the year. This helps us come up with 5680 companies within over seventy industry sectors[3]. Then we ran the descriptive on this data set of how many companies consist of each industry sector. And we selected the industries that contain more than 40 companies. This gives us a total of 5007 companies and this is our final data pool. The descriptive is as following:

		频率	百分比	有效百分比	累积百分比
有效	10	84	1.7	1.7	1.7
	13	163	3.3	3.3	4.9
	20	81	1.6	1.6	6.6
	27	45	.9	.9	7.4
	28	308	6.2	6.2	13.6
	33	52	1.0	1.0	14.6
	35	173	3.5	3.5	18.1
	36	312	6.2	6.2	24.3
	37	73	1.5	1.5	25.8
	38	203	4.1	4.1	29.8
	44	42	.8	.8	30.7
	48	124	2.5	2.5	33.2
	49	129	2.6	2.6	35.7
	50	68	1.4	1.4	37.1
	51	46	.9	.9	38.0
	58	48	1.0	1.0	39.0
	59	49	1.0	1.0	39.9
	60	421	8.4	8.4	48.4
	62	69	1.4	1.4	49.7
	63	124	2.5	2.5	52.2
	67	1710	34.2	34.2	86.4
	73	353	7.1	7.1	93.4
	80	53	1.1	1.1	94.5
	87	94	1.9	1.9	96.3
	99	183	3.7	3.7	100.0
	合计	5007	100.0	100.0	

3.1.1

The first column on the left are the SIC codes of different industries and the second column next to it shows the number of companies in that industry. The third column gives a sense of how many percent of the entire pool does in each industry construct. All the empirical evaluations are based on the data selected from the above data pool. The data pool contains a total of 5007 companies and each has 12 entries representing the monthly return of each stock[4].

SKEWNESS ANALYSIS ON TWO-ASSET PORTFOLIOS

After reading all the empirical studies on how the individual stocks' skewness level affect the portfolio return and risk, we decide to use a different method to also try to empirically test the effect of skewness on portfolio risk and return and try to grasp some sense in stock selections based on skewness.

According to Markowitz's modern portfolio theory, all portfolio valuations should start from the basic. This means that in order to look at the performance of a complicated multi-asset portfolio, we should first instead look at a simplified version with only two assets. This chapter will construct studies on two asset portfolios by incorporating skewness into modern portfolio theories and determine the effect of skewness on the efficient frontier and the optimal point on the frontier. We will first randomly select pairs of stocks and calculate the correlation coefficient, portfolio skewness and then calculate minimum variance points for each of the two asset portfolios. Among all the portfolios we construct, we will observe the differences of the efficient frontier between portfolios with the same correlation coefficient while different individual asset skewness level. The question we are interested in is: with the same correlation coefficient, does different skewness combinations shift or change the shape of the efficient frontier, and thereby shift the minimum variance, and if yes, how?

According to the literatures we gathered, when incorporating skewness in portfolio optimization, most models choose to try, under certain constraints, maximizing the skewness level in a portfolio. Many scholars as we mentioned in chapter two such as Arditti (1967) argued that theoretically, positive skewness is more preferred for not having extremely negative values. From all these previous evidences, this experiment is also trying to support previous studies from another facet. Therefore, before we go into the details, our assumption is that positive skewness should have a bigger impact on risk reduction and on the opposite, negative skewness should have a smaller impact or negative impact on risk reduction of the two-asset portfolio.

Adopting the Markowitz mean-variance model, we construct the mean variance model of two-asset portfolios. Suppose we have two risky assets and we know the means are $\overline{R1}$ and $\overline{R2}$, variances $\sigma_1^2$and σ_2^2, and a correlation ρ between the two. The weights of the two portfolios are w and (1-w), for $w \in [0,1]$.

Then the portfolio mean of this asset is:

$$\overline{R_p} = w * \overline{R_1} + (1-w) * \overline{R_2}.$$

The portfolio variance is:

$$\sigma_p^2 = w * \sigma_1^2 + (1-w)^2 * \sigma_2^2 + 2\rho * \sigma_1\sigma_2 * w(1-w)$$

As the weight w changes, we trace out a curve for this two-asset portfolio with the standard deviation σ on the x-axis and the mean return of the portfolio $\overline{R_p}$ on the y-axis. In our selected data set, which contains more than five thousand stocks with their monthly return on the year of 2011, we screened out the stocks that have significant skewness level and also the ones with skewness that is very close to 0. To quantify this criteria, we set the screening limits to stocks that have skewness level of 0, or greater than 1 or less than -1. The reason for us to not include moderate skewness levels (i.e (0,1) or (-1,0)) in this experiment is that we believe significant values may have more effects to the two-asset portfolio, which will be easier to observe. In addition, we assume that moderate skewness levels have the same direction of effect as its extreme cases. For example, we assume that holding all other constant, A_1 and A_2 are two stocks with the same return and standard deviation but A_1 has -1.2 skewness and A_2 is -0.3. A_1 and A_2 have the same correlation coefficient to B respectively. We then assume that A_1 and A_2 will have the same direction of effect on the two-asset portfolio and A_1's effect will be more significant. This selection does not deny that moderate skewness will have no effect or little effect to the portfolio, but only to help us observe the effect easier. This assumption works when the two assets have the same sign of skewness, i.e. they have both positive skewness levels or they both have negative skewness. This experiment does also include the possibility when the two stocks have the opposite sign, where one has positive skewness and the other has negative skewness. Intuitively, the result of this combination is ambiguous as one positive skewed and one negative skewed asset may shift the portfolio skewness somewhere between these two polar and through this test, we will try to discover the behavior of this combination as well.

From these selected stocks, we used SPSS to calculate a correlation matrix that describes the correlation between every two of the stocks[5]. We then selected the stock pairs that have correlation coefficient from 0.6 to 0.8 and -0.6 to -0.8. These stock pairs each form a two-asset portfolio we discussed early in this chapter. In order to see the effect of individual skewness on the risk and return of its two-asset portfolio, we control for the level of correlation coefficient, which due to heavy calculation and organization effort, is set as a specific interval rather than being categorized to different levels and analyzed respectively. In short, we will only pick the stock pairs that have quite significant correlation coefficient that is 0.6 and 0.8.

We then categorize these selected stock pairs into four different groups by the combination of its individual stock skewness. To make it clear, suppose we have 1000 stock pairs, which are also known as our two-asset portfolios and two individual stocks, A and B, form each of these portfolios. A and B both has a unique skewness level. Depending on the skewness combination of A and B, we constructed four categories:

i. Skewness of A and B are both greater than 1, denoted Pos & Pos

    The two-asset portfolios in this group are built by two stocks that both have skewness greater than 1.

ii. Skewness of A and B are both less than -1, denoted Neg & Neg

    The two-asset portfolios in this group are built by two stocks that both have skewness less than -1.

iii. Skewness of A and B are both 0, denoted Zero & Zero

The two-asset portfolios in this group are built by two stocks that both have skewness equal or very close to 0.

iv. Skewness of A and B are larger than +1 and less than -1 respectively, denoted Pos & Neg

Hence, we sort all the stock pairs we selected into two big categories (+0.6 to +0.8 and -0.6 to -0.8) and each category contains the four groups we stated above. For each portfolio in these groups, we calculate out the optimal point of the portfolio using minimum variance portfolio theory. "A portfolio of individually risky assets that, when taken together, result in the lowest possible risk level for the rate of expected return. Such a portfolio hedges each investment with an offsetting investment; the individual investor's choice on how much to offset investments depends on the level of risk and expected return he/she is willing to accept.[6] The investments in a minimum variance portfolio are individually riskier than the portfolio as a whole. The name of the term comes from how it is mathematically expressed in Markowitz Portfolio Theory we introduced in the previous chapter, in which volatility is used as a replacement for risk, and in which less variance in volatility correlates to less risk in an investment. Just as the name, minimum variance portfolio aims to calculate the optimal weight that can achieve the smallest portfolio variance. To minimize the risk is exactly what this paper is aiming to do so we adopted this portfolio construction method in doing two-asset portfolio analysis.

After massive computation, for each group we have 10 portfolios with their minimum variance optimized. Then for each of these portfolios, we calculated the risk reduction level in terms of percentage. The risk reduction rate is quantified by comparing the optimized portfolio's standard deviation to the smaller standard deviation of the two assets, which can be noted as the following:

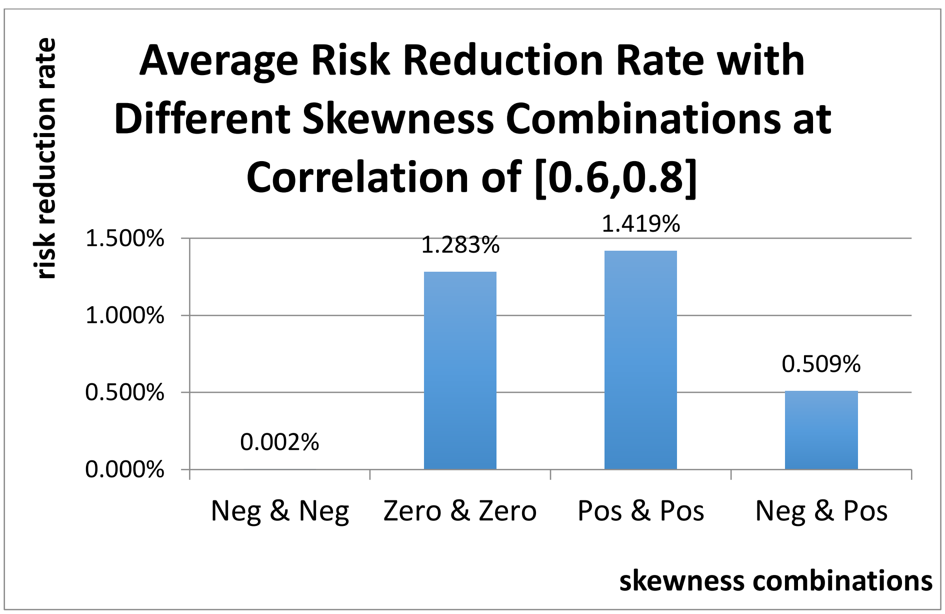
Risk Reduction Rate= $[S_p – \text{Min}(S_1, S_2)]/ \text{Min}(S_1, S_2)$

In the above equation, S_p is the minimum variance portfolio's standard deviation and S_1, S_2 are the standard deviation of the two assets. The risk reduction rate can give us a sense of how much the standard deviation is reduced in each case when forming a portfolio and help us compare which skewness combination on average has the highest risk reduction rate. For correlation coefficient at 0.6 to 0.8, we have four groups and also for correlation coefficient at -0.6 to -0.8, we have the same four groups. Each group as we have picked has more than ten two-asset portfolios and we finally calculated the arithmetic mean of the risk reduction rate for each group. The result is as following:

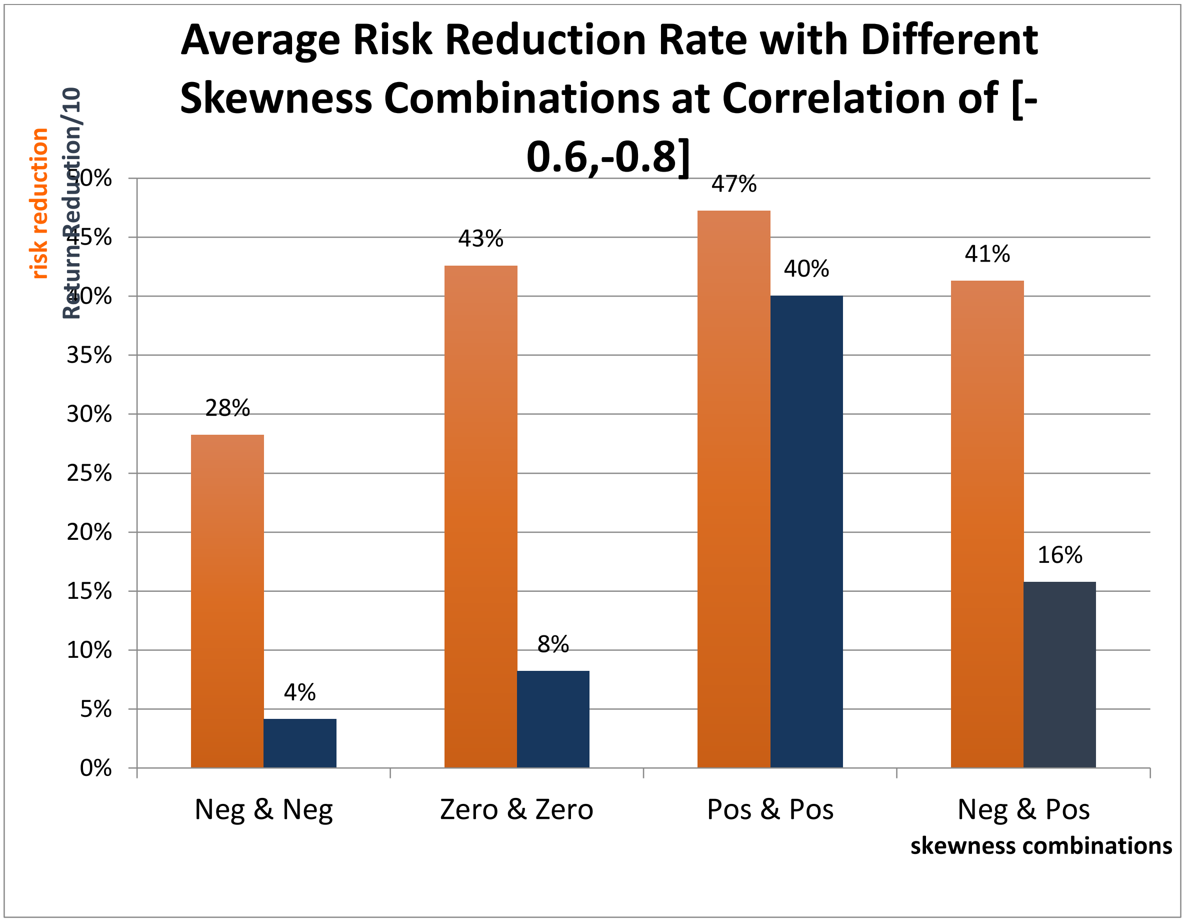
Average Risk Reduction Rate				
Groups	Neg & Neg	Zero & Zero	Pos & Pos	Neg & Pos
Correlation [-0.6, -0.8]	28%	43%	47%	41%
Correlation [0.6, 0.8]	0.002%	1.283%	1.419%	0.509%

4.1.2

As we can see in the above graph, we also calculated the return reduction rate when the correlation coefficient is negative. When correlation coefficient is positive and especially at a high level of larger than 0.6, we know that the portfolio is very hard to diversify as the two assets in this kind of portfolio are highly correlated. Normally since the goal is to diversify risk, we do not consider involving too many highly correlated assets. Thus the study of the positive correlation coefficient is just to see if the result is consistent with the result from the negative correlation coefficient and see if there are any similar observations. This is the reason we focus more on negative correlation coefficient possibilities rather than positive.



4.1.3



4.1.4

The first and the most important observation is that no matter the correlation coefficient is negative or positive, the combination of two negative skewed assets (group Neg & Neg) has the lowest risk reduction rate. When the two assets are positively correlated, there is almost no risk reduction for two negative skewed assets. Although as we discussed that positively correlated equities almost do not reduce any risk when put into a portfolio together, we still find that there is some risk reduction at 1% in some group we constructed. Comparing to this, the risk reduction rate of group Neg & Neg in [0.6, 0.8] is only at 0.005%, which is almost zero. This shows us the probability that when both assets are negatively skewed, the risk reduction is probably low. Luckily, we find the same result in the category where correlation coefficient is [-0.6, -0.8]. From the graph above we can see that the average risk reduction rate for negatively correlated portfolios that consist two negatively skewed stocks is 28%, a percentage more than 30% lower than the second lowest reduction rate, which is at 41%. This supports our assumption that the combination with two negatively skewed assets has the lowest risk reduction rate.

On the opposite, the best combination is hard to be decided as the risk reduction rate for groups Zero & Zero, Pos & Pos and Neg & Pos at both negative and positive correlation coefficient are all clustered in the same range (Positive correlation is at 1.283%, 1.419% and 0.509% respectively and negative correlation groups are at 43%, 47% and 41%), which can only be distinguished from the reduction performance of the Neg & Neg group, which has significant lower performances. From only this sample report, we can see that no matter the correlation is positive or negative, the none-skewed group and the group with both positive skewness has the highest risk reduction rate.

Another exciting observation is that we found the rank of the risk reduction rate is exactly the same for the two categories when the correlation is positive or negative. The group that has the highest risk reduction rate in both cases is Pos & Pos, where the portfolios in that group are contained by two positive skewed equities, then Zero & Zero follows and then comes the Neg & Pos group and at last is the group Neg & Neg.

On the level of two-asset portfolios, we confirm previous studies on skewness preference based on our empirical experiment and we find that negative skewness is not conducive in reducing the risk level of portfolio and thus negatively skewed assets are less preferable in the selection of two-asset portfolios. Two-asset portfolios that contain two negatively skewed assets have the smallest effect in reducing risk and the risk reduction rate for portfolios that consist one negatively skewed asset is also lower than those that do not have any negatively skewed asset.

SKEWNESS ON RISK REDUCTION RATE IN MULTI-ASSET PORTFOLIOS

From our study on portfolio selection, one of the most classic methods is the traditional selection method. According to Markowitz's modern portfolio theories, in order to diversify the portfolio risk, one should pick elements that are less correlated. He suggested a positive relation between risk and portfolio correlation, which indicates that the higher correlation between the selected equities is, the higher risk this portfolio will have. Hence the traditional selection corresponds to investor's most common sense that in order to diversify portfolio risk, one should select stocks that are less correlated so that one does not "put all the eggs in one basket." And the "baskets" on the basic level of understanding is obviously industry sectors. Before the emergence of more advanced portfolio selection methods, portfolio risk diversification is always equivalent to choosing stocks that are in different industry sectors. Many at first believed that different industries, intuitively, should have their individual behavior, and hence have different performances. This makes many scholars believe that between most industries the correlation coefficient is low. Despite the fact that some industries are actually moving in the same direction or sometimes highly correlated, choosing stocks in different industries is always considered to be the easiest and the fastest way to diversify risk. Note that this method is not the best portfolio selection method as we have pointed out that certain industries are highly correlated and selecting stocks that are in different industries does not mean that these stocks are less correlated. For example, if we picked stocks in the oil industry and also stocks in transportation industry such as airline companies or car companies, we will most likely not to be able to diversify too much risk because even though they are in different industries, one industry solely relies on the other since oil is the primary power for transportation and any change in the oil industry will definitely affect the transportation industry as well. No matter what, to first have a look at the possible relation between skewness and portfolio risk, this section will adopt this method and select stocks from all the twenty-five different industries of the data set introduced in chapter three.

According to our dataset description (see table 3.1.1), we neglected industries that have less than 40 companies and it returns to us 25 different industries and a total of more than five thousand companies. From each of these industries, we randomly select one stock and as a result we will have a portfolio that contains twenty-five stocks. We then repeat this selection procedure and finally construct 40 portfolios using this method.

For each portfolio, using their monthly returns on the year of 2011, we calculate the variables we need for our regression using equal weight (each company in the portfolio takes 1/25 of the portfolio). There are a total of six variables generated from the data. The variable names and descriptions are as the following:

	Variable Abbreviation	Variable Name
Dependent Variable	riskreducrt	Risk Reduction Rate
Independent Variable	portskw	Portfolio Skewness
Control Variables	portreturn	Portfolio Return
	avrcorr	Average Correlation
	negskw	Number of negatively skewed asset in %
	posskw	Number of positively skewed asset in %

5.1.1

Riskreducrt = (average standard deviation – portfolio standard deviation) / average standard deviation

Using the data we gathered and import them into STATA, we first run an OLS using the simple regression function trying to have a brief glance at our data and possible problems we are facing. The following is our simple regression results of portfolio skewness on portfolio reduction rate with all the control variables we select:

. reg riskreducrt portskw portreturn avrcorr negskw posskw

Source	SS	df	MS
Model	.068129307	5	.013625861
Residual	.035654209	34	.001048653
Total	.103783517	39	.002661116

Number of obs	=	40
F(5, 34)	=	12.99
Prob > F	=	0.0000
R-squared	=	0.6565
Adj R-squared	=	0.6059
Root MSE	=	.03238

riskreducrt	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
portskw	-.0315055	.012136	-2.60	0.014	-.0561689	-.0068422
portreturn	.0567247	.7161016	0.08	0.937	-1.398569	1.512018
avrcorr	-.5626669	.0895122	-6.29	0.000	-.7445776	-.3807562
negskw	.0565572	.1098867	0.51	0.610	-.1667594	.2798737
posskw	.044399	.0887605	0.50	0.620	-.135984	.2247819
_cons	.5751221	.0839774	6.85	0.000	.4044594	.7457847

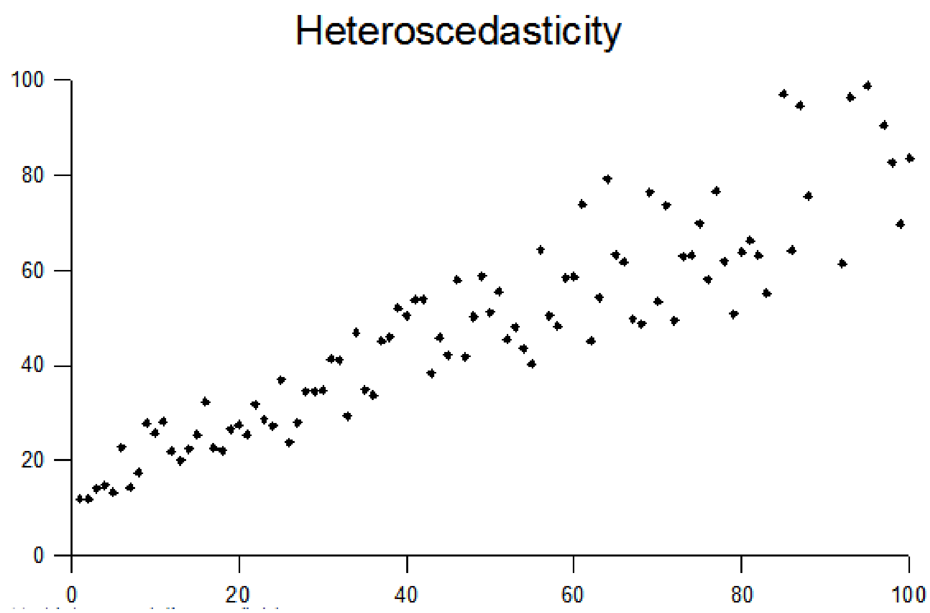
5.1.3

From the above result we can see that the p-value for our independent variable is 0.014, which is smaller than the critical value 0.05. This means that the test result is significant at this point.

We now move one to test for multicolinearity in the data. We adopted the test of variance inflation factor for multicollinearity (VIF) to check if any of the variables we used are highly correlated. Multicollinearity is a statistical phenomenon in which two or more predictor variables in a multiple regression model are highly correlated, meaning that one can be linearly predicted from the others. If this is the case the estimates of the coefficient may change erratically in response to small changes in the model or the data. However "multicollinearity does not reduce the predictive power or reliability of the model as a whole, at least within the sample data themselves; it only affects calculations regarding individual predictors."8 This means that a multiple regression model with correlated predictors can indicate how well the entire bundle of predictors predicts the targeting variable but it may not give valid results about any individual predictor, or about which predictors are redundant with respect to others.

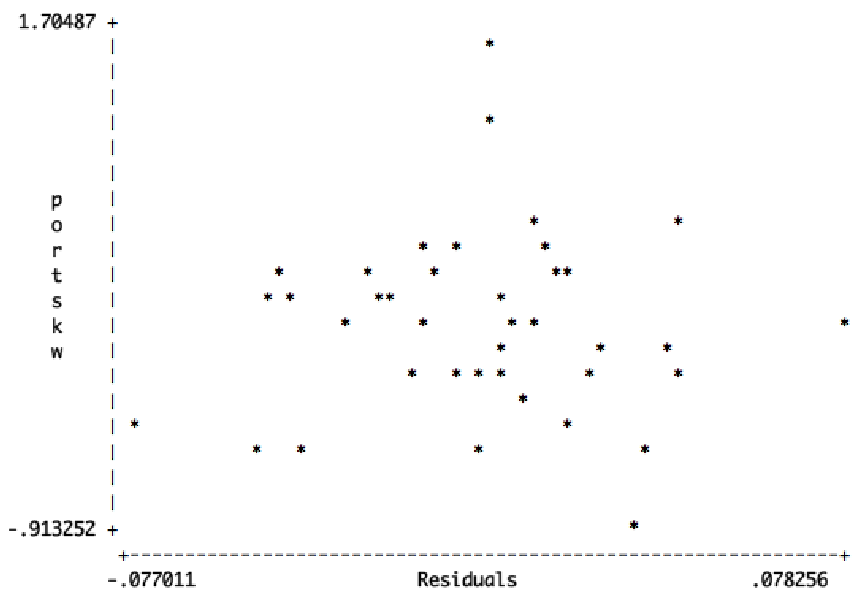
Thereis no variable that has a VIF value even over 5. This means that there is no problem of multicolinearity in this regression model.

Now since we have data of all different portfolios, we are actually dealing with cross section data. Cross-sectional data refers to data collected by observing many subjects (such as individuals, firms or countries/ regions) at the same point of time, or without regard to differences in time. Analysis of cross-sectional data usually consists of comparing the differences among the subjects. Our data set fits this profile as we are comparing the differences among the 40 portfolios. The most common problem we face in cross-section data is heteroskedasticity. The basic idea is that if the data set is heteroskedstic, it means that the error of the dependent variable does not have a constant variance. A most common example is as the following:



5.1.5

From this we can see that a typical case of heteroscedasticity has the following distribution of error, where the variance of the error is not stable. We need to test our sample to see if there is any significant level of heteroskedasticity. Therefore, we plot out the relation of our independent variable and our error term. The graph is as following:



5.1.6

From this graph we do not see a very obvious trace of heteroskedasticity as the trend of the error is vague. Thus instead, we use Cook & Weisberg's test on heteroskedasticity to see if there is any alternate result that can be clearer. The Prob> Chi^2 is not very significant. This tells us that this data set is mostly homoscedastic.

For the relation between portfolio skewness and the risk reduction, the research result in fairly interesting. We can see that the p-value for this independent variable is also close to zero. This tells us that in our data set, the level of portfolio skewness does affect the portfolio risk level and hence as Mr. Markowitz and other scholars predicted, higher movements should indeed be taken into account. However, what astounded us the most is that the coefficient of the independent variable is actually negative. This result is exactly the opposite of our assumption that we made from the findings in two-asset portfolios. We assumed that the relation between the portfolio skewness and portfolio risk reduction rate should be positive, meaning that the more positive the portfolio's skewness is, the higher rate of risk reduction one should have. However, we have to say that in this year of 2011 and under the traditional portfolio selection method, the relation between portfolio skewness and portfolio risk reduction rate is negative.

CONCLUSION

Even though as many scholars have agreed that positive skewness is always preferred, the empirical testing result generated in this paper proves that this stock selection criteria needs more consideration. The result shows us counter examples in which negative skewness is actually preferred in risk reduction. Both selection methods indicate that an increase in either portfolio skewness or number of positively skewed assets in the portfolio will actually decrease the risk reduction rate and hence give us a less optimized portfolio. These findings rejected the previous study results that positive skewness is always preferred for investor who seeks to lower their portfolio risk. We should certainly not generalize such assertion since we only focused on one specific year of the stock market and we only adopted two stock selection methods. However, this study still has its significant implication that when using portfolio optimization it is not always the case that positive skewness should be desired. Sometimes, negative skewness may result in a better improvement in portfolio risk reduction. This discovery rings a bell to the current investors that when including skewness in portfolio optimization, one should at first run tests on the stock market and see its behavior in at least the previous year in order to have a better understanding of whether positive or negative skewness is preferred in the portfolio risk reduction at the moment. And then, after a short study, the investor can optimize his portfolio using his model and findings on the skewness effects. Therefore, the paper concludes that it is not always true that positive skewness is preferred in risk minimization. Sometimes, as our experiment result tells us, more negative skewness can actually decrease the portfolio risk. Investors' discretions are highly advised when trying to incorporate skewness into portfolio optimization, as the effect of skewness is not always positive.

ENDNOTES

1. Nobel Lecture, H. Markowitz, Dec 7 1990, Baruch College
2. Beedles, W. L., and M. A. Simkowitz. "Morphology of Asset Asymmetry." Journal of Business Research, 8 (Dec. 1980), 457-468.
3. A complete description of the data is available from the author
4. Sample data is available from the author
5. Sample chart is available from the authors.
6. Jason Hsu (2011) " Quantitative Asset Management Notes on Minimum Variance Portfolios"
7. Gujarati, Damodar. "Multicollinearity: what happens if the regressors are correlated?". Basic Econometrics (4th ed.). McGraw−Hill. pp. 363–363.

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