Production Functions in Major League Baseball: A Star Input Method

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ABSTRACT

This is an empirical investigation of the winning function in major league baseball. As more star players are added to a team by signing free agents, the winning percent for that team first increases at an increasing rate but eventually at a decreasing rate. In Rodney Fort's *Sports Economics* textbook, this theoretical production curve is shown graphically for major league baseball but is not estimated with a regression model. This is surprising because a production function is not only important on its own, but it also plays a significant role in supporting the shapes of subsequent cost and profit functions in Fort's text. In this paper I empirically estimate the winning function by identifying and summing the star hitters and pitchers by team, and then correlating these numbers to team win percents. I define a star player as one with a slugging percentage or on-base percentage one standard deviation above the league average for starting players. Star pitchers and relievers will be similarly defined. I aggregate ten years of league cross-sections (2002-11), and investigate several alternative functional forms. The key issue is whether the estimated parameters of a given functional form support the theoretical curvature shown in *Sports Economics.*

INTRODUCTION

Rodney Fort's leading sports economics textbook shows a theoretical production function featuring star players as team inputs and wins as team output (Fort, 2011, p. 100). Using hypothetical data for Major League Baseball, its shape shows increasing returns and then diminishing returns to the number of stars. This underlying production function plays a key role in his development of long run cost curves for professional sports teams. But there is no formal empirical model testing the shape of the production shown in the text. This is surprising because a production function is not only important on its own, but it also plays a significant role in supporting the shapes of subsequent cost and profit functions in Fort's text. In this paper I will estimate the winning function for professional baseball using a unique measure of star players.

BACKGROUND

There will be a distinction drawn between two types of sports production functions. The first is a "team-based" production function because it arises out of a desire to empirically test hypotheses about the role different team-based inputs play in determining team success. These production functions are common in the literature, going back to 1974 (Scully), in which team winning percent (W) is specified as a multivariate

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function W = f(team hitting, team pitching, team standings, end-of-season team contention variables). Building on Scully, several authors use regression models to investigate the marginal revenue product of hitting versus pitching (MacDonald and Reynolds, 1994), the marginal product of batting average versus slugging average (Zech, 1981), the effect of managerial ability on winning (Porter and Scully, 1982), the effect of team racial composition on winning (Hanssen, 1998), and the evidence for Moneyball (Hakes and Sauer, 2006; Farrar and Bruggink, 2011). Winning percent functions based on team inputs have evolved over the years in baseball, including the use of newly created statistics replacing more traditional measures of hitting and pitching. Similar production functions are investigated in basketball (Chatterjee, Campbell, and Wisemen, 1994), football (Hadley, Poitras, and Ruggiero, 2000), soccer (Carmichael, Thomas, and Ward, 2001) and hockey (Leard and Doyle, 2011), to test hypotheses on home field advantage, team efficiency, and the effectiveness of coaching ability.

I call the second type of sports production function the "player-based" production function. This provides the two-dimensional production function found in Fort's sports economics text. It is a special version of the standard production function found in any introductory economics textbook. For sports, however, team output is winning percent, and the number of *star* players is on the horizontal axis. The reason for star players as the labor input rather than simply the number of players is that all teams put nine players on the field, so there is no variation from team to team. In Fort's *Sports Economics* the production curve has the same shape as in the introductory economics texts. Yet other than a pedagogical article (Bruggink, 1993), there is no literature or empirical estimation on the player-based production functions for sports. This player-based production function is the focus of this study, and the theory behind the shape of the curve will be discussed in the next section.

THEORY FOR THE PLAYER-BASED PRODUCTION FUNCTION

The theoretical production function for winning uses star players as its variable input. As more star players are added by signing free agents, total product (winning percentage) increases first at an increasing rate but eventually at a decreasing rate. Why does winning function first increase at an increasing rate? One star player does little to help a horrendous team. The opposing team could always just pitch around him in critical situations. And if the lone star did get a good pitch there frequently are not any teammates on base. A second star adds an incrementally *higher* amount to team winning percentages because when the lineup contains two star hitters, opposing pitchers are less likely to pitch around the first one or walk him intentionally. However, if more and more stars are added and the team's winning percent is already high, the increments to winning diminish. When a team already has star players at every position and the pitching staff has six first-rate starters and several ace relievers, the rules of baseball constrain the marginal contributions of additional stars (they cannot all play at once).

It is a peculiarity of sports that the production function has a long run upper limit. Wins cannot exceed the fixed number of games for a season. As a more practical approach Fort puts the upper limit at 70% wins. This approximates the highest win percent ever attained in modern baseball. The win percents associated with Fort's production function are taken from a table in his textbook (p. 100), and in Table 1 the change in percent wins of each star player is also calculated. In Fort's hypothetical production function, the win percent of a team with no star players is 42%. Each additional star player adds higher amounts to percent wins until the seventh star player. At this point the increases to talent decline. Fort attributes this decline to "Limits to managing more and more stars must be the explanation for diminishing returns." (p.101), although additional reasons are provided below. However this explanation neglects the fact that only nine players are on the field at a given time. This fixed input compromises the assumptions of a long run production function, but works as a better explanation for the diminishing returns than manager inefficiency.

Table 1 Fort's Hypothetical Winning Percent for MLB

stars	**win pct**	**change**
0	42	-
1	42.9	0.9
2	44.8	1.9
3	46.6	1.8
4	49	2.4
5	52.8	3.8
6	59.8	7
7	64.6	4.8
8	68	3.4
9	69.5	1.5
10	70	0.5

Fort's win production curve is not technically a long run curve because in sports some inputs remain fixed by the rules of the game: the number of players on the field and on the roster, the number of games in a season, the length of the games, etc. The consequence is a maximum on input and output possibilities, and therefore diminishing returns remains as an outcome in the long run.

The functional form is not specified, but Fort describes the curve as one that "increases at a decreasing rate" after seven stars are on one team (Fort, p. 100). The earlier part of the curve shows increasing at an increasing rate. Theory does not suggest the degree of the polynomial that will best describe the shape of this production function. For this reason the following theoretical shapes will be included for comparison, here $winpct_{it}$ is the winning percent for team i in year t and $tstars_{it}$ is the number of stars on team i in year t:

1. $winpct_{it} = B_0 + B_1\, tstars_{it} + B_2\, tstars_{it}^2 + B_3\, tstars_{it}^3$
2. $winpct_{it} = B_0 + B_1\, tstars_{it} + B_2\, tstars_{it}^2 + B_3\, tstars_{it}^3 + B_4\, tstars_{it}^4$
3. $winpct_{it} = B_0 + B_1\, tstars_{it} + B_2\, tstars_{it}^2 + B_3\, tstars_{it}^3 + B_4\, tstars_{it}^4 + B_5\, tstars_{it}^5$

Each of the above polynomials can provide a curve that is increasing at an increasing rate, and then increasing at a decreasing rate, depending on the magnitudes and signs of the coefficients.

DATA SELECTION PLAYER-BASED PRODUCTION FUNCTION

In the previous section the theoretical production function provided by Fort sets the stage for the rest of this paper. He defends the hypothetical data to generate the curve as

> "…an intuitive snapshot of the relationship between adding stars and winning; it is not the result of any more extensive analysis than you just have read."(p.100)

This study will empirically estimate the production function for baseball by identifying and summing the star hitters and pitchers by team, and then correlating these numbers to team win percents. Star players are not defined by Fort but described as "a cut above the average player" (p. 100). I believe a player with *one standard deviation above average league performance* captures this simple definition of a star. This ensures a limited number of players designated as stars (17% of the rosters), while marking the player as one of statistical distinction. This definition is somewhat unique in that it does not rely on official league designations to identify stars ("All-Stars"), nor does it recognize prior performances or reputation. A discussion on the sample design is next.

First I identify a pool of active players over the years 2002-2011. The stars chosen from this pool will have a minimum number of at-bats (AB) or innings pitched (IP) beyond what it takes to initially qualify for this pool, and will have an important performance statistic that is one deviation from the weighted average in the pool.

Players with a more than trivial participation in a given season form this initial pool. For hitters in this initial pool, this will be at least 100AB in a season, resulting in about half of all hitters listed on *mlb.com* for a given season. For starting pitchers I take players with at least 40 IP and for relievers at least 30 IP. These arbitrary cutoffs again take about half of the starting pitchers and half the relievers listed on mlb.com as having participated in a given season. Players designated as stars, however, must have more participation than just these active players. Star hitters must have at least 300 AB, starting pitchers must have at least 80 IP and relievers must have at least 40 IP to be selected from the initial pool. This second set of cut-offs is also arbitrary.

Players designated as star hitters will be those with an on-base-percentage (OBP) or slugging percentage (SLG) one standard deviation higher than the pool average in their league. Star pitchers will be those with an *opposing* OBP or SLG one standard deviation below the pool average in their league. Both the means and the standard deviations are weighted by either at-bats or innngs pitched. To summarize, for purposes of this paper a star player in a given season is defined by one of the following:

1) a hitter with at least 300 AB and an OBP or SLG one standard deviation above the weighted average for all players in that league with at least 100 AB.

2) a starting pitcher with at least 80 IP and an opposing OBP or SLG one standard deviation below the weighted average for all starting pitchers in that league with at least 40 IP.
3) A relieving pitcher with at least 40 IP and an opposing OBP or SLG one standard deviation below the weighted average for all relievers in that league with at least 30 IP.

The consequences of this definition include:

1) a player considered a star over his career might not be a star every season.
2) a player who performs a key role in his team's success but does not have the sufficient AB or IP will not be acknowledged.
3) a player with an impressive number of runs batted in, home runs, stolen bases, etc., might not be identified as a star.
4) an admirable performance by a player might still miss the one standard deviation cut-off for OBP or SLG.
5) stars with performances two standard deviations above the league average are counted the same as players with just barely over one standard deviation.

Although these consequences are disappointing they are not devastating to my investigation.

Table 2 shows the summary statistics for the total number of stars in the sample, as well as the star players disaggregated by type. Based on the last ten years, the mean stars per team is a little more than four. This breaks down to nearly two hitters, slightly over one starting pitcher, and little over one reliever.

Table 2 Sample Statistics—Stars per Team

Variable	Obs	Mean	Std. Dev.	Min	Max
Star hitters	300	1.89	1.329051	0	7
Star starters	300	1.05	0.9507082	0	5
Star relievers	300	1.33	1.133748	0	6
Total stars	300	4.27	2.246265	0	12

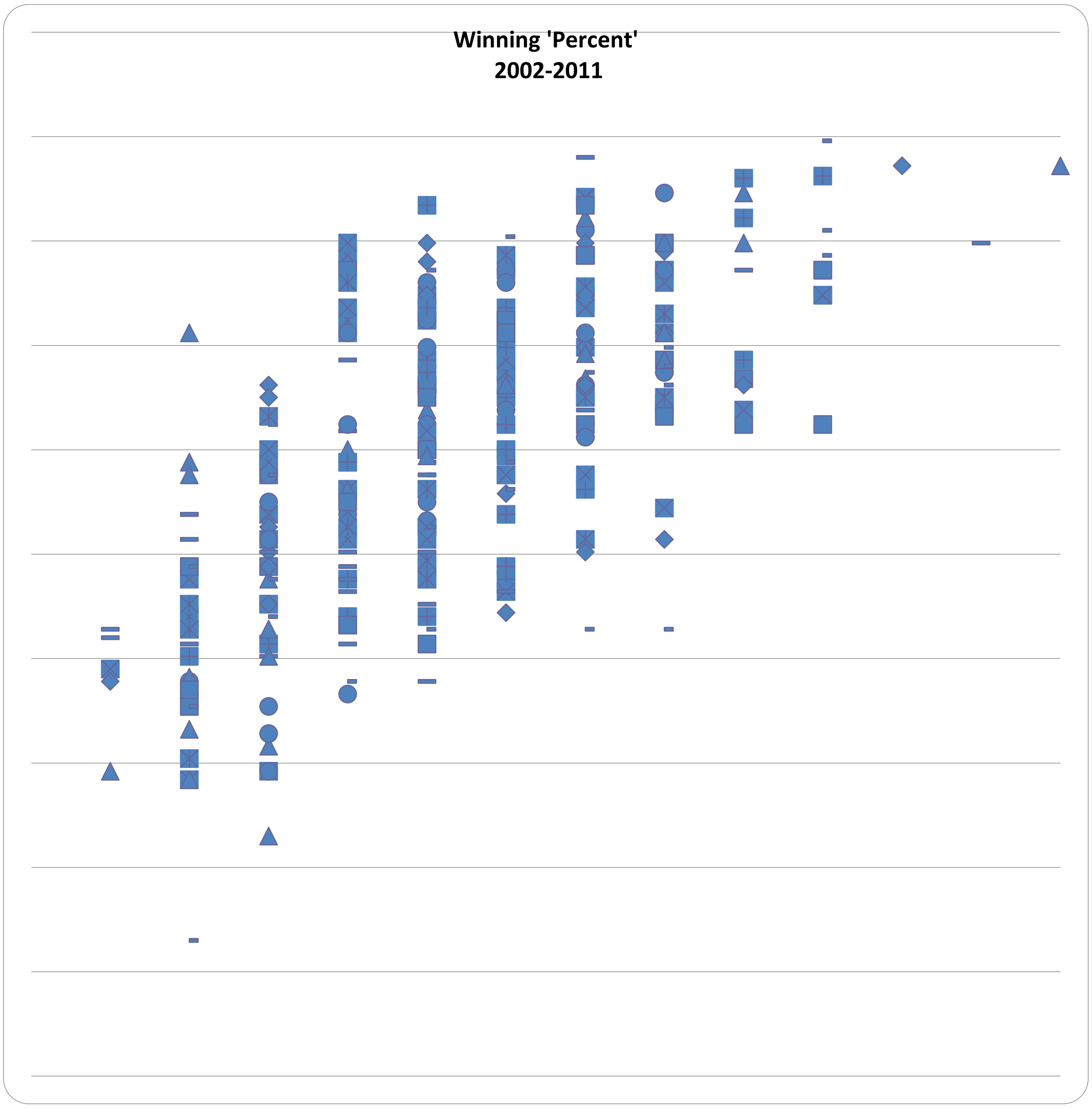
The scatter diagram below (Figure 2) illustrates the data from which the curvature fits will be estimated. Three observations are relevant here. First, there is considerable variation in percent wins for each number of star players per team. This warns of a lack of precision in estimating any functional form. Second, beyond seven stars, the number of observations falls dramatically. This signals particular challenges in using a regression model to verify Fort's hypothetical shape for teams with eight or more stars. This means I will focus on teams with up to seven stars only. Third, there are only six outlying data points in the sample.

RESULTS FOR THE PLAYER-BASED PRODUCTION FUNCTION

The regression results for the three specifications discussed earlier are shown in Table 3. The variable tstars is the total number of star hitters and star pitchers on each team. The dependent variable pctwin is the team percent wins expressed on a scale from 0 to 1000, where 1000 represents 100 percent (for example 620 is 62% wins). This is done for convenience and as a concession to the vernacular. Because multicollinearity is an issue with the tstars power terms, I will not rely on the t-coefficients to determine the case for the verification of Fort's production function.

The overall R^2 fit of roughly 50% and the typical error of 51 (5.1 percentage points) reflects the wide variation in wins for each star level seen in the scatter diagram. This sobering result hampers the type of specificity that is the goal of this research. I begin with one overall declaration: all three specifications show diminishing returns of wins with respect to the number of star players, but only the degree five regression supports Fort's particular shape of increasing, then decreasing returns.

Figure 2 Scatter Diagram for 2002-2011



Number of Star Players

Table 3 Regression Results for the Player-Based Winning Function

Variable	Degree 3 Coeffcient	Degree 4 Coefficient	Degree 5 Coefficient
Intercept	374.8	381.4	392.8
tstars	43.2	33.8	6.48
tstars^2	-3.34	0.556	16.8
tstars^3	0.127	-0.416	-4.29
tstars^4		0.0244	0.416
tstars^5			-0.0139
RMSE	51.2	51.3	51.3
R-sq	0.495	0.495	0.497
Adj R-sq	0.490	0.488	0.489

In Figure 3, I plot the degree five regression over a typical range of seven star players. At first one sees growing increases in percent wins, but with the fourth star player, the increases decline. The size of these increases is revealed in Table 4. The first three star players add 1.94%, 3.27 %, and 3.31% increases in percent wins, followed by a 2.72% increase for star player four. Increases begin to sharply fall off with players five to seven (2.0%, 1.49%, and 1.34%, respectively).

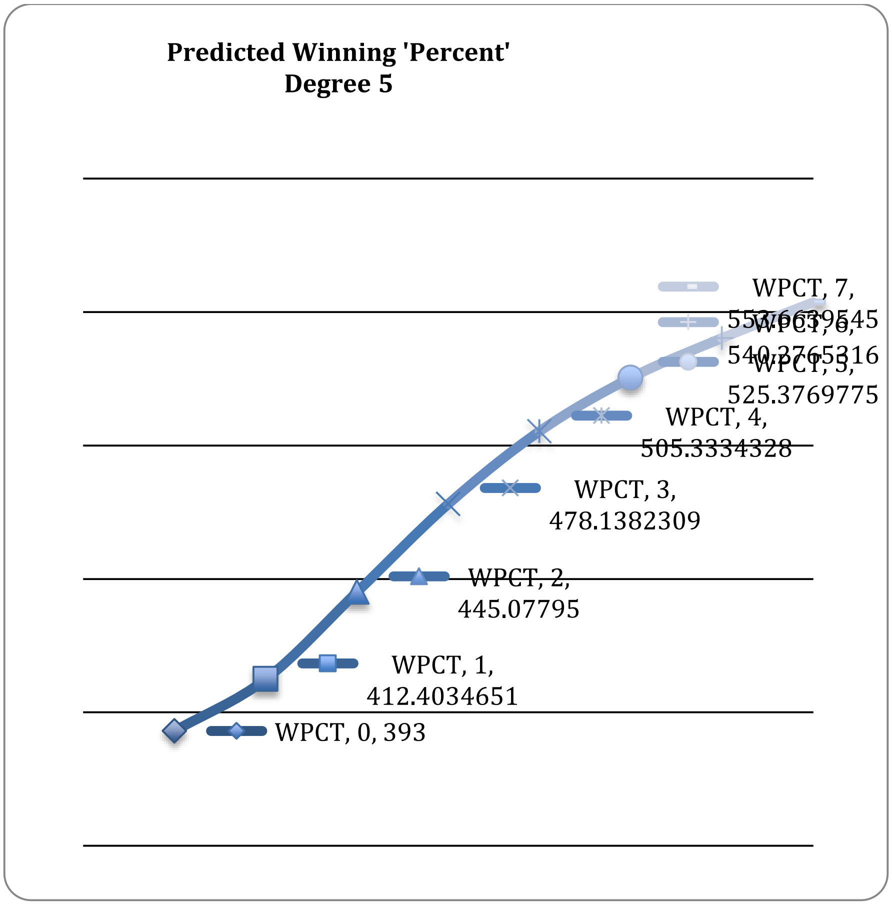
The empirical results for the cubic and degree four equations fail to support the particular shape of Fort's textbook production function. Diminishing returns begin immediately. Although these production curves cannot be ruled out as being true winning functions, the conclusions will only discuss the winning function for the degree five equation.

CONCLUSION

There is empirical support for the general shape of Rodney Fort's baseball production function shown in his popular sports economics textbook. One key issue is whether the rate of increase rises at first before falling. In the *Sports Economics* text the rate of increase starts to decline with the addition of the seventh star player. My empirical results shows the rate of increase declines starting with the fourth star player, and the increases fall sharply thereafter. Only the fifth degree polynomial equation supports Fort's shape, but the regression for the cubic and fourth degree regressions have the essentially the same measure of statistical fit. In other words the data does not reveal a superior fit between the degree three, four, or five functional forms.

To my knowledge until now no one has attempted to statistically measure a player-based production function in any sport. I present detailed empirical evidence that support of Fort's hypothetical production function for baseball, although my results suggest some alteration in the shape to more closely align his hypothetical curve with my empirical curve. This has important implications in using these empirical outcomes to predict the benefits of acquiring additional star players and in generating long run cost curves for teams. Finally, this approach has potential for a new avenue of research in not only baseball but in all competitive sports.

Figure 3 Predicted Winning ‘Percent’ Winning Function from Degree 5 Regression



Number of Star Players

Table 4 Estimated Winning ‘Percent’ from Degree 5 Regression
(divide by 10 for actual winning percent)

stars	‘win pct’	change
0	393.0	-
1	412.4	19.4
2	445.1	32.7
3	478.1	33.1
4	505.3	27.2
5	525.4	20.0
6	540.3	14.9
7	553.7	13.4

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