Dancing with the Bees: Follow-the-Leader Market Dynamics with Robot Firms

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ABSTRACT

This paper reviews a market experiments designed to capture an information feedback process, and test its effectiveness in leading a market to a profit maximizing market equilibrium when agents do not engage in explicit profit-maximizing decision-making. Instead robot firms adapt to the performance of the top firm in the market, and also to market conditions, in updating their decisions on price and production. We find that the simple information feed-back process leads to profit maximizing market equilibrium in a competitive market with no profit maximization built into the firm-level decision process.

INTRODUCTION

This paper reviews a market experiments designed to capture an information feedback process, and test its effectiveness in leading a market to a profit maximizing market equilibrium when agents do not engage in explicit profit-maximizing decision-making. Instead robot firms adapt to the performance of the top firm in the market, and also to market conditions, in updating their decisions on price and production.

The particular information feedback process tested here has attributes of the foraging process in which a hive of bees can locate a specific and critical source of food as described by Thomas Seeley in *The Wisdom of the Hive*. In this process there is no discussion among the individual bees about where to look. Instead a host of scout bees is sent out in every direction. Upon their return, those that discover any food engage in a special “waggle” dance. The better the source, the better the dance. Others are attracted to a variety of the best dancers and follow the leaders to their sources. After a few iterations “disciples” switch to the best dancers and eventually the single best food source is selected by the hive. This process requires a variety of mutually independent observations, and an information feedback process in which hive members follow those who have chanced upon the best food source. These are some of the characteristics of group decision making discussed by James Surowieki in *The Wisdom of Crowds*.

In the information feedback process tested in this study, in Period 1 we apply a normally distributed random variable on the common decisions of Period 0. This generates a wide array of possible decisions on the part of firms in each market. Clearly some of these decision sets fail miserably, but some work very well. As the market process progresses, there is a continual tuning process that improves upon the previous period’s results as long as there continues to be variation in decisions offered across firms.

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The success of this process requires a sufficient array of possibilities generated in Period 1, and in subsequent periods and also an efficient information feedback process to bring others along. We find that the simple information feed-back process leads to Market Equilibrium in a competitive market. These results are interesting. Economists often face critiques about the role profit maximization plays in our economic models. Business firms rarely use marginal revenue or marginal cost in their actual decision-making. Results of experiments reported here suggest that even when that is true, a simple follow-the leader information feed-back process can lead market participants to results that look as if they were based on profit maximizing behavior when in fact they were not.

The Market Process

The market period corresponds to a business quarter. decisions are made each period by each firm in the market. The marketing decision is price per unit, and the production decision is labor hours for the quarter to be used with a specified (fixed) plant size. The market process entails a beginning "trial" period in which all decisions are identical across participating firms. This is followed by Period 1, in which firms choose their own price and labor hours without much direction of any kind. This involves adding a normally distributed random component across firms to the Period 0 decisions. In Periods 2 onward, firms review the results of the previous period and adapt according to a logistic process that includes a normally distributed random component and information on decisions of top firms, market conditions and individual firm's inventory. There is no maximizing behavior built into this process of any kind. Production is Cobb-Douglas in form:

$$(1) \qquad TP_i = GL_i^a K^b$$

Where: TP_i is firm *i*'s total production in the current period; G is a constant; L_i is firm *i*'s labor hours in the current period; K is units of Capital (fixed at 4), and *a* and *b* are parameters.

G = 6100, K =4, a = 0.25, and b = 1.04.

This gives rise to total, average and marginal costs as follows:

$$(2) \quad TC_i = W\left(\frac{1}{GK^b}\right)^{\frac{1}{a}} TP_i^{\frac{1}{a}} + RK$$

$$(3) \quad ATC_i = W\left(\frac{1}{GK^b}\right)^{\frac{1}{a}} TP_i^{\left(\frac{1}{a}-1\right)} + \frac{RK}{TP_i}$$

$$(4) \quad MC_i = \tfrac{1}{a} W\left(\frac{1}{GK^b}\right)^{\frac{1}{a}} TP_i^{\left(\frac{1}{a}-1\right)}$$

Where: TP_i is total production in the current period by firm *I; TC_i* is total cost for firm *i* in producing TP_i; ATC_i is average total cost for firm *i* in producing TP_i; MC_i is marginal cost for firm *i* in producing TP_i; W is the wage per hour; and *R* is the implicit rental rate of capital.

In this market, W = $52.00, and R = $252,000. There are 36 firms in the market. The market supply function from (4) is:

(5) $$Q_S = N\left(\frac{P}{\frac{1}{a}W\left(\frac{1}{GK^b}\right)^{\frac{1}{a}}}\right)^{\left(\frac{1}{\frac{1}{a}-1}\right)}$$

Where: Q_s is market quantity supplied; *N* is the number of firms in the market; and *P* is market average price.

Market demand is Cobb-Douglas/log linear:

(6) $$Q_D = CP^{e_p}$$

Where: Q_D is Market Quantity Demanded; *P* is the market average price; *C* is a constant; and e_P is market price elasticity of demand.

In this market, *C* = 81,823,500, and e_P is -1.1

Period 0

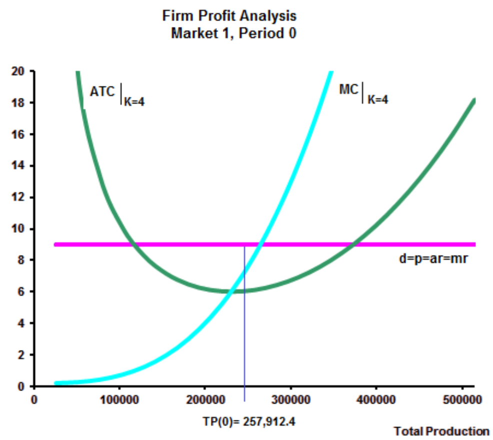
In Period 0, firm decisions are set as follows:

$p_{i,0} = \$9.00$ and $l_{i,0} = 10000$ for all *i*.

Where: $p_{i,0}$ is the price of firm *i* in period 0; and $l_{i,0}$ is the labor hours of firm *i* in period 0.

Using (1) and l_i = 10,000, each firm produces TP_i = 257,912.4 units. Unconstrained profit analysis for a firm in Period 0 with model assumptions and Period 0 decisions is illustrated in Figure 1.

Figure 1



Market goods available for sale in Period 0 is 9,284,846.4. Market demand using (6) and p_i = \$9.00 is 7,298,124.91. In the competitive market, the good sold is perfectly homogeneous, so with a common price, this demand is split evenly across firms so that in Period 0, each firm's quantity demanded is 202,725.7 and its sales are therefore restricted to this figure. (See Appendix 1, Period 0). This means each firm has 55,186.71 in unsold goods at the end of the period. Since inventory is assumed perishable, this is lost before commencing with Period 1.

Period 1

In Period 1, the "hive" sends the "bees" out to scout for a new "food source".

Firm decisions are as follows:

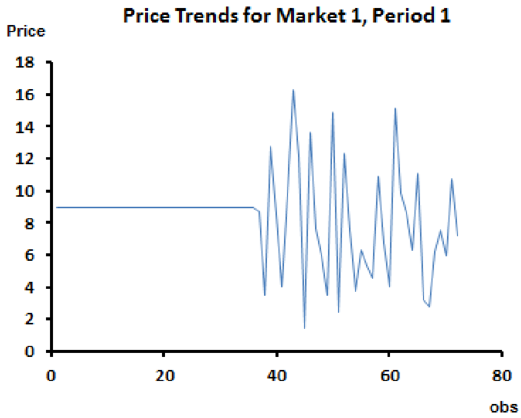
(7) $p_{i,1} = p_{i,0} + \varepsilon_1$

(8) $l_{i,1} = l_{i,0} + \varepsilon_2$

Where: ε_1 is a random normal disturbance on price; ε_2 is a random normal disturbance on hours; and $p_{i,0}$ = \$9.00, and $l_{i,0}$ = 10,000.

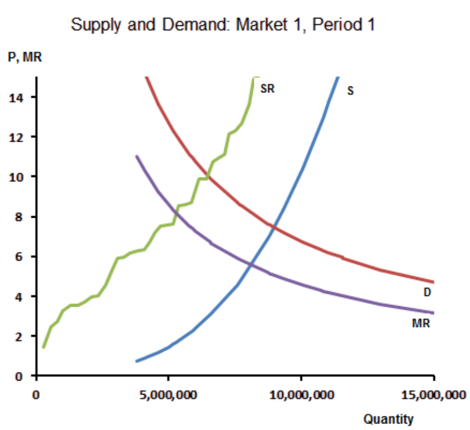
The resulting price time series for Period 1, are shown in Figure 2:

Figure 2



In the competitive market, the good sold is perfectly homogeneous. Buyers always purchase the cheapest offerings first. Firms' prices are therefore sorted from lowest to highest along with quantities offered for sale. This is illustrated in Figure 2 as the Supply Response (SR). Market demand from (6) is also plotted in Figure 3 and labeled (D). Firms offering goods to the northeast of the intersection between SR and D do not sell. In Period 1 the highest price that sold was \$10.00. Firms offering goods for prices higher than \$10 did not sell in Period 1. With perishable inventory these goods are lost prior to Period 2. This process implies that an individual firm in the competitive market will face a perfectly elastic demand at is chosen price as long as it can sell P <= \$10 in Period 1. Otherwise its demand vanishes. With this setup, firms with a price closest to the highest price that sells and also with production closest to the level where the firm's price equals its marginal cost will be the firms with highest profits.

Figure 3



In Period 1 it is clear that firms are generally not profit maximizing and are generally over-pricing their goods.

Period 2 and Beyond

In periods 2 and beyond "bees" return to the "hive" and "dance". Those with the most "flashy" dances attract other "deciple bees". In Periods 2 and onward, therefore, robot firms adjust their prices and labor hours. These adjustments respond to information feedback from the previous period, and focus on three factors:

a) the prices and output of the top ranked firm,
b) the firm's goods available in comparison with its share of market demand, and
c) Market Goods available compared with Market demand.

The specific form of these three information feed-back relations is a logistic process in each case as follows:

(9a) $$\left(p_{top}\right)_{i,t} = p_{i,t-1} + p_{i,t-1}\left(1 - \frac{p_{i,t-1}}{p_{top,t-1}}\right)\varepsilon_3$$

Where:

$(p_{top})_{i,t}$ is firm *i*'s price adjustment tied to the top firm's previous period price; $p_{i,t-1}$ is firm *i*'s price in the previous period; $p_{top,t}$ is the top firm's price in the previous period; and ε_3 is a normally distributed random variable, with a mean of 1.

So if a firm's price was less than the top firm's price in the previous period, $(p_{top})_{i,t}$ is adjusted upward. If it was higher, it is adjusted downward. Since this is a logistic form, the adjustment asymptotically diminishes the closer a firm's price was to the top firms' price last period. Note that this also diminishes the impact of ε_3, the normally distributed random variable.

(9b) $$\left(p_{fg}\right)_{i,t} = p_{i,t-1} + p_{i,t-1}\left(1 - \frac{fg_{i,t-1}}{qd_{i,t-1}}\right)\varepsilon_4$$

Where:

$(p_{fg})_{i,t}$ is firm *i*'s price adjustment tied to firm *i*'s inventory in the previous period; $p_{i,t-1}$ is firm *i*'s price in the previous period; $fg_{i,t-1}$ is firm *i*'s finished goods available for sale in the previous period; $qd_{i,t-1}$ is firm i's share of quantity demanded in the previous period; and ε_4 is a normally distributed random variable, with a mean of 1.

So if a firm's goods available was less than its share of market demand in the previous period, $(p_{fg})_{i,t}$ is adjusted upward. If its goods available was greater, it is adjusted downward. Since this is a logistic form, the adjustment asymptotically diminishes the closer a firm's share of demand approached its goods available last period. Again, this diminishes the impact of ε_4, the normally distributed random variable.

(9c) $$\left(p_{MG}\right)_{i,t} = p_{i,t-1} + p_{i,t-1}\left(1 - \frac{MG_{t-1}}{Qd_{t-1}}\right)\varepsilon_5$$

Where:

$(p_{MG})_{i,t}$ is firm i's price adjustment tied to market conditions in the previous period; $p_{i,t\text{-}1}$ is firm i's price in the previous period; $MG_{,t\text{-}1}$ is market finished goods available for sale in the previous period; $QD_{i,t\text{-}1}$ is market quantity demanded in the previous period, and ε_5 is a normally distributed random variable, with a mean of 1.

So if there was a surplus of goods available compared with market quantity demanded in the previous period, $(p_{MG})_{i,t}$ is adjusted upward. If there was a shortage, it is adjusted downward. Since this is a logistic form, the adjustment asymptotically diminishes as surpluses or shortages diminish. Again, this diminishes the impact of ε_5, the normally distributed random variable.

These are each considered by the robot firms using a weighting scheme as follows:

(10) $$p_{i,t} = w^{p}_{top}\left(p_{top}\right)_{i,t} + w^{p}_{fg}\left(p_{fg}\right)_{i,t} + w^{p}_{MG}\left(p_{MG}\right)_{i,t}$$

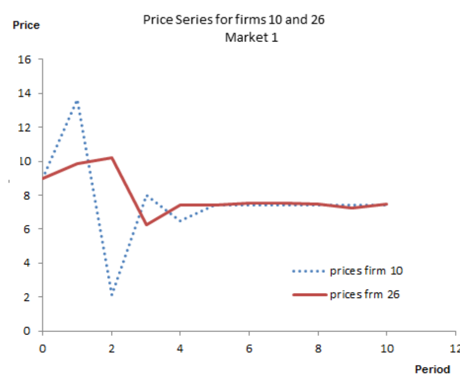
(11) $$w^{p}_{top} + w^{p}_{fg} + w^{p}_{MG} = 1$$

For the competitive market run, weights are set as follows:

$w^{p}_{top} = 0.7$, $w^{p}_{fg} = 0$, and $w^{p}_{MG} = 0.3$

According to this weighting scheme, firms mostly focus on the top firms previous price and output decisions, along with some consideration of market quantity demanded as compared with the amount of goods actually offered for sale in the previous period. In the competitive case, buyers always choose the lowest price that has not sold out. As they go from the current best price to the next best, firms sell out and the effective quantity demanded at each firm will equal the firm's goods available for sale, unless the firm has priced itself out of the market. For this reason, in the competitive case we set $w^{p}_{fg} = 0$, but adjust $p_{i,t}$ from (10) by -.05$p_{i,t\text{-}1.}$ Figure 4 illustrates this price adjustment process for Firms 10 and 26 in the competitive market experiment over the market process, Periods 0-10:

Figure 4



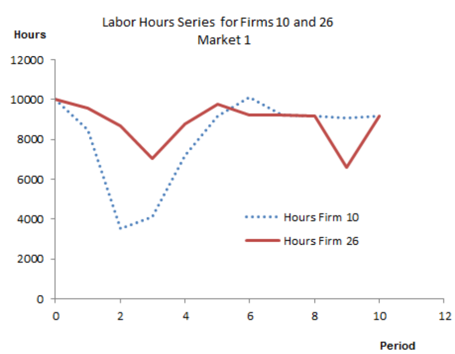
A similar process is used for generating the labor hours decisions in periods 2 and onward in the competitive market experiment.

(12a) $$(l)_{i,t} = l_{i,t-1} + l_{i,t-1}\left(1 - \frac{l_{i,t-1}}{l_{top,t-1}}\right)\varepsilon_6$$

Where: $(l)_{i,t}$ is firm *i*'s labor hours in period *t*; $l_{i,t-1}$ is firm *i*'s labor hours in the previous period; $l_{top,t}$ is the top firm's labor hours in the previous period; and ε_6 is a normally distributed random variable, with a mean of 1.

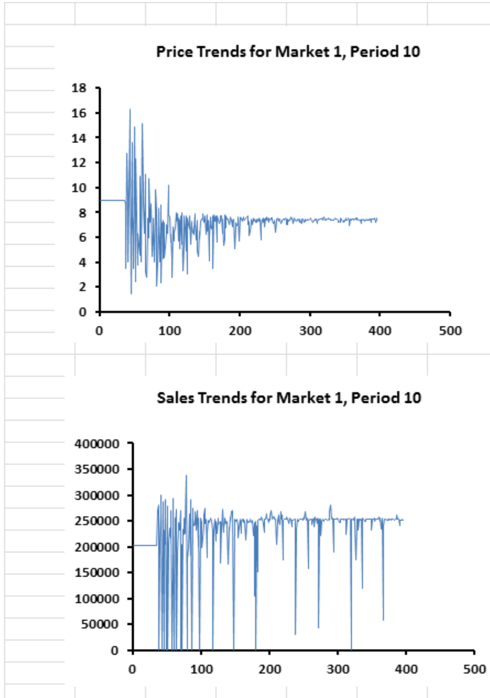
So if a firm's hours were less than the top firm's hours in the previous period, they are adjusted upward. If they were higher, they are adjusted downward. Since this is a logistic form, the adjustment asymptotically diminishes the closer a firm's hours were to the top firms' hours last period. Note that this also diminishes the impact of ε_6, the normally distributed random variable. Firms only consider the top firm's production decision in adjusting their labor hours each period, so a weighting function for labor ours like () for price is not used. The process is illustrated for firms 10 and 26 in Figure 5.

Figure 5



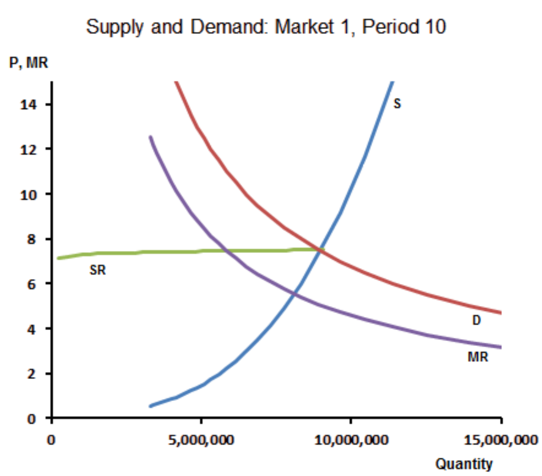
The time series of prices and output for the entire competitive market run are shown in Figure 6.

Figure 6



These results suggest the market is moving toward equilibrium by Period 10.

Figure 7



The process of adjustment that takes place in Market 1 from Period 2-10 results in a dramatic change in the character of the supply response, (SR) in Figure 6 as compared with Figure 2. Note that by Period 10, nearly all firms are charging the equilibrium market price and as a group they are offering the market equilibrium quantity for sale. Buyers are in turn purchasing this quantity. So as a group, sellers have adjusted their decisions so that the profit maximizing market equilibrium is achieved. This is especially interesting because sellers have no information on marginal cost or market supply. They simply make adjustments tied to basic information on top firm production and pricing, along with information on market conditions and inventories, and price adjustment process is not explicitly connected to the labor hours adjustment process.

Conclusions

We find that the simple information feed-back process leads to Market Equilibrium in a competitive market and Nash equilibria in imperfectly competitive markets. Clearly, this process is capable of leading to sub-optimal equilibria traps, but testing up to this writing appears to indicate that these usually are surprisingly close to the optimal equilibrium solution.

These results are interesting. Economists often face critiques about the role profit maximization plays in our economic models. Business firms rarely use marginal revenue or marginal cost in their actual decision-making. Results of experiments reported here suggest that even when that is true, a simple follow-the leader information feed-back process can lead market participants to results that look as if they were based on profit maximizing behavior when in fact they were not.

Appendix 1: Model Parameters

Distribution of Weights for Price and Labor Hour Adjustments

	competition	mon. comp.	oligopoly	monopoly
w^{p}_{top}	0.7	0.7	0.7	0.7
w^{p}_{fg}	0	0.15	0.15	0.3
w^{p}_{MG}	0.3	0.15	0.15	0
w^{l}_{top}	1	1	1	1
w^{l}_{fg}	0	0	0	0
w^{l}_{MG}	0	0	0	0

Parameters

		competition	mon. comp.	oligopoly	monopoly
Production:	G	6100	6100	6100	6100
	K	4	4	4	4
	a	0.25	0.25	0.25	0.25
	b	1.04	1.04	1.04	1.04
Cost:	W	52	52	52	52
	R	252000	252000	252000	252000
Firms:		36	36	5	36
Market Demand:	C	81823500	81823500	11364375	81823500
	e_p	-1.1	-1.1	-1.1	-1.1
Firm Demand:	c		2272875	2272875	2272875
	α		-3.1	-3.1	-1.1
	β		2	2	0