Time-Changed Lévy Jump Processes with GARCH model on Reverse Convertibles

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ABSTRACT

For decades, financial institutions have been very motivated in creating structured high-yield financial products, especially in the economic environment of lower interest rates. Reverse Convertible Notes (RCNs) are the type of financial instruments, which in recent years - first in Europe and then in the U.S. – have become highly desirable financial structured products. They are complex financial structured products because they are neither plain bonds nor stocks. Instead, they are structured products embedding equity options, which involve a significant amount of asset returns' uncertainty. Given this fact, pricing of Reverse Convertible Notes becomes a really big challenge, where both the general Black-Scholes option pricing model and the Compound Poisson Jump model which are designed to catch large crashes, are not suitable in valuing these kinds of products. In this paper, we propose a new asset-pricing framework for Reverse Convertible Notes by extending the pure Brownian increments to Lévy Jump risks for the underlying stock returns movements. Our framework deals with time-changing volatilities of stock options with Lévy Jump processes by considering the stocks' infinite-jump possibilities. We then use a discrete-time GARCH with time-changed dynamics Lévy Jump processes in order to derive the assets' valuations. The results from our new model are close to the market's valuations, especially with the Normal-Inverse-Gaussian model of the Lévy Jump family.

Keywords: Lévy Jump Process, Fourier transforms, exotic options, reverse convertible, stochastic volatility, GARCH

1. INTRODUCTION

This paper deals with a pricing methodology for structured products and more precisely, for the valuation of Reverse Convertible Notes (bonds). For decades, investors have been diligently searching for high-yield financial investment instruments in an economic environment of low interest rates. Thus, the increasing demand for high-yield products has given financial institutions plenty of motivation and opportunity to create financial structured products. Since the financial crisis and economic doldrums of 2008, the Untied States short-term interest rates have remained at the low level of 0-1% for a long and extended period of time. Chart-1 in the Appendix, which demonstrates the historical yield of the 13-week United States Treasury Bills, confirms this. Our data further indicates that during the mid-2011, the rates even became negative. These negative yields mean that investors would lose some of their investments by investing the assets in real term. This low-yield economic environment motivated many investors to

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look for those risky, high-yield driven structured products. A structured product is a financial asset in which the attributes of several other financial assets are combined in one. A Reverse Convertible Note (RCN) or Reverse Convertible Bond (RCB) is one of these structured products. It is a combination of a fixed-income note and an equity option. It's also an over-the-counter product. In general, a RCN is a short-term investment vehicle with a one or two year maturity date. However, the real attraction of a RCN is that it offers generous yields. Some of the products even offer a double-digit yield. This is highly desirable for such a short investment period, even when compared to 10-year U.S. Treasury bonds, which offer much less yield during the same period. Table-1 in the Appendix shows a list of sample Reverse Convertible Notes traded in the markets as of May 2011. From the details of this list, we can see the attractiveness of the coupon rates of reverse convertibles. Based on the data we obtained that there are more than 94% of RCNs have coupon rates with two digits during the period of year 2008 through 2012. In addition, since the U.S. Central Bank (the Federal Reserve) started the program of open market purchasing of United States Treasuries, in order to stimulate the U.S. economy--also called Quantitative Easing (QE) -- and decided to purchase $40 billion mortgage backed securities on a monthly basis, beginning in the summer of 2012, the Treasury yields are going to be driven even lower across the Treasury yield curve. These kinds of economic environments have pushed investors to take risks in chasing high yields and more broadly demanding risky products, such as RCNs. Despite the attractive yield an RCN can provide, however, its risks and complexities can be a real challenge to most investors, and especially to retail investors. Chart-6 in the Appendix shows the changes in an RCN due to the changes of its underlying stock. When the underlying stock of a RCN drops to the pre-determined trigger value, the RCN will loss a portion or all principal. Therefore, an RCN is not principal protected, in general. An investor could, therefore, lose his/her entire investment, even though the note's issuer does not default. This is certainly different than a conventional note or bond. A RCN pays an unconditional coupon rate in exchange for forgoing any appreciation in the underlying shares. Over the term of the note, if the underlying shares have traded at or below the pre-determine ratio of their initial face value, the notes would be forced to be redeemed in shares instead of getting the principal amount in cash. In other words, there is an option built into this investment vehicle. Comparatively, a RCN is like a convertible bond in that the bond can be converted to underlying shares. However, unlike the convertible bond, a RCN gives the conversion right to the issuer -- not the note holder. At maturity, when the note's underlying stock price is lower than its pre-determined stock price (trigger price), the note can be either paid at face value or be converted into equity, as the issuer decides. In fact, we might consider a RCN as embedding a "put" option, held by the note issuer, who will exercise the option if the underlying stock price goes below the pre-determined level during the life of the contract. A put option gives the holder a right but not the obligation to sell the underlying stock with predetermined price at a future time. As a result, while investors are lured by the attractive yield offered by a RCN, they must bear in mind that they simultaneously write a put option, which could hurt them considerably when the stock price drops below the trigger level in the future. The reason an investor is able to get such a high coupon rate from a RCN is that he/she sold the ‘put’ option to the issuer at the time of purchasing the security. The premium of the put option is the primary source of an RCN’s high yield. Since the equity option is embedded in the note, the risk behaviors of the note are quite different from those of traditional fixed-income investment instrument, which might be very difficult for retail investors to comprehend, since the risks of RCNs are not linear because of the exotic options embedded in the products. It’s not uncommon that some RCN investors were surprised to find out that they lost the face values of the notes, even when the issuer is not involved in a bankruptcy. As previously discussed, a RCN is a structured product with an exotic option embedded in the asset. More precisely, most RCN deals are constructed with a barrier put option(s). A barrier option is one whose payoff is determined based on whether the price of the underlying asset reaches the barrier (pre-determined price level) during the life of the option. A down-and-in barrier put option’s payoff is zero when the underlying value is above the barrier. A down-and-in put option plus a down–and-out put option should equal a plain vanilla put option. As a result, the value of a down-and-in put option should be less than the value of the corresponding plain vanilla put option. Because a RCN is embedded with a down-and-in (knock-in) put option, the RCN’s price movements are closely correlated to the underlying stock’s volatility. In fact, the barrier options embedded in RCNs increase the complexity of the assets valuation. With a built-in knock-in put option, a RCN investor may lose the principal of his/her investment if the underlying stock price drops significantly. Therefore, this paper is to discover the risks associated with these products by providing a suitable valuation methodology for RCN products.

The rest of this paper is organized as follows: section 2 will provide the literature review. Section 3 introduces our valuation methodologies. Section 4 explains estimation processes and results. Section 5 draws the conclusions.

2. LITERATURE REVIEW

Traditional option valuation concerns itself with the classic Black-Scholes option pricing model, which is based on the assumptions that an asset’s returns follow Geometric Brownian motion and have constant stock volatilities. The assumption of stock return following Brownian motion has become a benchmark processes for asset’s derivative pricing models. However, in practice, we see that stock returns do involve jump processes and time-changed volatilities. As a result, real world stock returns involve much higher volatility than the Brownian motion suggested, as demonstrated in Chart-2 and Chart-3 in the Appendix. The difference is that the data in Chart-3 shows the real value of the S&P 500 index obtained from the markets, while, Chart-2 illustrates the value of the S&P 500 index is derived from the Black-Sholes pricing model. Clearly, we can see that the data in Chart-3 (real market data) is more volatile and involves more frequent jumps than the ones shown on Chart-2. Chart-4 shows a single stock’s volatilities and its price variation over a relatively long period of time. Similar to Chart-3, Chart-4 also indicates that stock return moves in a way of more frequent jumps. Chart-5 demonstrates the S&P 500 Index’s Implied volatilities (VIX); the volatilities of low-risk single stock; and the volatilities of high-risk single stock, all of which clearly indicate that the volatilities of stock return are not constant over time. These significant discrepancies have led researchers to revisit the geometric Brownian motion assumption. Because asset returns display this phenomenon -- a) returns jump over time; b) returns are stochastic; and c) returns and their volatilities are correlated -- the latest research literature proposes alternative pricing methodologies, such as Poisson Jump Process and Lèvy Jump Process, which can better capture the jump features of stock returns. Other literature has addressed the issues of a market's possibilities of large swings by adding Poisson jumps to Brownian distributions – one example being Merton's Jump Models. The purpose in adding Poisson jumps in the asset-pricing model is to model the large crash of the financial markets, or individual stock's swing movements, such as jump-to-default risks. However, other researchers, such as Carr and Wu (2004) and Li, Wells and Yu (2011) documented that there is evidence of small jumps that cannot be modeled using Poisson jumps. This evidence led to a new flood of research literature on developing methodologies to implement the models with those small jumps over the time, such as infinite-Lèvy jump processes. The models considering Lèvy jumps allow for more flexibility of asset pricing, especially for pricing options. The Lèvy jump process, named after the French mathematician Paul Lèvy, is a stochastic process with independent stationary increments. Many researches since Lèvy have been implementing this process to value derivatives. Barndorf-Neilsen (1995) first used inverse Gaussian type Lèvy jump process to model log returns of stocks. Madan, et al (1998), and Carr and Wu (2002) are among the few who proposed the methodologies of option pricing by considering return's distributions with jump processes. However, they used a continue time format to value European-style options. Because Lèvy jump is a better representation of the real world stock return movements, in this paper, we incorporate this process to evaluate RCNs – which some researchers have tried to evaluate in the past. For example, Wilkens and Röder (2003) used European - style put options to valuate the equity component of RCNs by assuming the underlying stock follows (GBM) and its volatilities are stochastic. They found that the RCNs were underpriced. We believe that Lèvy processes are more suitable than GBM and the general Gaussian driven processes, in evaluating a structured derivative, because it considers the stock return's jumps, skewness and excess Kurtosis etc. Deng, et al (2012) used a Variance Gamma type of Lèvy jump model to evaluate RCNs and concluded that RCNs are overpriced. However, a Variance Gamma approach is not the best approach in valuing a stock option, according to the research of Ornthanalai (2011). In his research, Ornthanalai tested five types of Lèvy jump models -- including Variance Gamma model -- by utilizing a huge database of on index options and stock returns over more than a ten-year time period. He generated these time-changed models with an affine GARCH that are easy to implement. His research indicated that the risk premium of infinite-activity jumps "significantly" dominated that of the Brownian increments and therefore, infinite-jump, rather than Brownian increments, should be the default option pricing model. Furthermore, from his research results, he noted that Normal Inverse Gaussian type Lèvy jump with GARCH model (NIG-GARCH) indicated the best results among those five jump processes models when he compared log likelihood values. In fact, the NIG Lèvy jump distribution is the only member of the family of general hyperbolic distributions to have closed convolution. Because of the facts mentioned above, in this paper we propose a new asset pricing framework for RCNs and other similar structured products by considering: 1) a stock returns' Lèvy jump processes with the time-changed GARCH model; and 2) pricing exotic options with the return processes in 1).

3. METHODOLOGY AND DATA

In our RCN pricing framework, we first establish a Normal-Inverse Gaussian (NIG) Lévy jump process with affine GARCH model. Then we built an exotic option - pricing model. Our approach is relatively easier to implement in a practical way. By using affine GARCH approach in a discreet time format, it's relatively easier to handle the return volatilities. Due to the complexity of a RCN product and in order to simplify the analysis, we can consider the structured product as an investment portfolio with the following two assets:

(1) Long -- a corporate bond;

(2) Short -- an exotic put option.

The issuer of the bond and the underlying stock of the option refer two different entities. The valuation of the proposed portfolio is:

$$RCN_t(B_t, f_t) = B_t(r_t, t) - f_t^I(S_t, r_t, K, H, t) \tag{1}$$

where, RCN_t is the value of a reverse convertible note at time t. B_t is the value of a corporate bond at time t. It's a function of interest rate r_t (it's risk-free rate plus the issuer's credit risk premium). f_t^I is the value of a exotic put option at time t. It's a function of underlying stock S, the strike price of the option K, and the barrier of the option H. The negative sign of the second term in equation (1) indicates that an investor sold the option to the issuer.

In the following subsections, we focus on deriving an exotic option valuation in order to pricing RCN. As we discussed above, we propose a new asset pricing methodology by introducing a probability distribution with a GARCH model:

3.1: LÉVY JUMP MODEL

Assume Y, the log returns of a stock, follows Lévy jump processes. A Lévy jump process is a probability model for an unpredictable measurement with right continuous and left limit $Y = \{Y_t\}_{t \geq 1}$ and it is a stochastic process under probability space (Ω) and filtration and Y_0=0. Y has stationary increments*. The increments are independent. Y_t changes in such a way that the changes of the measurement in disjoint time intervals of equal duration, $Y_{t_i+\Delta t} - Y_{t_i}$ and $Y_{t_j+\Delta t} - Y_{t_j}$ are i.i.d. $\forall$i, j. Let $X_{t+s} = Y_{t+s} - Y_t$, S>0. We refer Y_t as a Lévy process and X_t as the Lévy innovation. The characteristic function of Y_t describes the distribution of each increment is given by $\psi_{x.}$ Many statisticians in all kind of fields use the methodology of generalized Fourier transform of a Lévy innovation when, the density function unavailable. According to probability theory, a characteristic function of any real value random variable completely defines its probability distribution. The characteristic function is the inverse Fourier transform of the random variable's probability density function. Therefore, even though we aren't clear about the density function of Lévy innovation, we can still derive the security's value through inverse Fourier transform technique if we know its characteristic function.

Definition: for a scalar random variable Y, the characteristic function is the expected value of e^{itY}, where i is imaginary unit with $i=\sqrt{-1}$ and t∈R is the argument of the characteristic function. Mathematically,

$$\psi_Y(t)=E[e^{itY}]=\int_{-\infty}^{\infty}e^{ity}dF_Y(y)=\int_{-\infty}^{\infty}e^{ity}f_Y(y)dy, \qquad \psi_Y: R\in C.$$

where, F_Y is the cumulative distribution function of Y. f is variable y's density function.

According to Lévy Khintchine Theorm**, the characteristic function of Y_t has the form

$$F_{Y_t}(u)\equiv E[e^{iuY_t}]=e^{-t\psi_Y(u)}, \quad t\geq 0, \qquad (2)$$

where $\psi_Y(u)$ is defined as exponent characteristic function. u is in the complex domain D such that (2) is well defined.

Let $Y_t=\log\frac{S_t}{S_0}$ presenting stock's log return during time [0,t]. For next one-period Lévy innovation conditioning at t, x_{t+1}, the generalized Fourier transform is

$$\Psi_x(u;t,t+1)\equiv E[e^{ux_{t+1}}]=e^{\psi_x(u;t,t+1)} \qquad (3)$$

where, $\psi_x(u;\, t,\, t+1)$ is conditional cumulated exponent of x_{t+1}. Comparing equation (2) and (3), we know that the exponent of the transform in Lévy innovation is not linear in time as in the Lévy process. Therefore, we cannot use Carr, et al (2004)'s approach to implement random time changing technique. Instead, we must apply the approach of Ornthanalai (2011) because Lévy innovations are assumed to be time homogeneous in one of their parameters:

$$\psi_x(u;t,t+1)=h\eta_x(u), \qquad (4)$$

where, h is the homogeneous parameter of x_{t+1} and it is independent from $\eta_x(u)$ which is the coefficient in the cumulated exponent. At this point, in equation (4), the cumulated exponent of Lévy innovation x_{t+1} is linear in h. We can rewrite equation (4) as following without lose any value because h_{t+1} is known as time t:

$$\psi_x(u;t,t+1)=h_{t+1}\eta_x(u), \qquad (5)$$

Now, equation (5) becomes a dynamic model that can generate the similar effect of a random time change on Lévy process. And h_{t+1} changes through heteroskedastic that can be used as GARCH approach.

3.2: LÉVY GARCH MODEL

Now we can use discreet time to model an asset's return. Under a risk-neutral measure, an asset's return in an affine GARCH (1,1) is given+

$$R_{t+1} = r_{t+1} - d_{t+1} + B_{t+1} + J_{t+1} - (0.5h_{B,t+1} + \eta_{J,t+1}h_{J,t+1}), \quad (6)$$

$$h_{B,t+1} = \gamma + \beta h_{B,t} + \frac{\alpha(B_t - ch_{B,t})^2}{h_{B,t}}$$

where, R_{t+1} is asset's return at time of t+1. r_{t+1} and d_{t+1} are risk-free rate and dividend yield at time t+1, respectively. B_{t+1} and J_{t+1} are the shocks at time t+1. B_{t+1} follows Brownian motion and B_{t+1} ~ Normal (0, $h_{B,t+1}$). J_{t+1} is the Lévy jump innovation factor under risk-neutral measure. $\eta_{J,t+1}$ is convexity adjustments at time t+1. $\eta_{J,t+1}h_{J,t+1}$ is the cumulated exponent of the jump innovation.

3.3: BARRIER OPTION VALUATION

As we discussed before, a RCN has two components -- a bond and an exotic put option. The value of the RCN is the value combination of both the bond and the embedded put option. The value of a bond is straightforward. The difficulty of a RCN valuation involves evaluating the knock-in put option. In fact, it's relatively easier to evaluate a knock-out put option than a knock-in put option. Therefore, we value knock-out put option first and then we use the relationship that a knock-in put option is equal to a plain vanilla put option minus a knock-out put option. Under the condition $Q \equiv \{H < S_{t+\tau} \mid t < \tau < T\}$, we have the knocked-out put option value:

$$f^o{}_t = E_t^Q\{e^{-\int_t^{t+\tau}(r_s - d_s + \nu_s)dS} \max[(H - S_{t+\tau}), 0]\} \quad (7)$$

where, H is the barrier level of the option. $E_t\{.\}$ denotes the expectation operator at a risk-neutral world measure at time t. v is the risk premium of the stock. If at time t+τ, H≥S, the knock-out put option, $f^o_{t+\tau}$, becomes worthless. As we discussed before, the value of a knock-in option is the difference between the value of a plain vanilla put option and the value of a knock-out put option, i.e.

$$f_t^I \equiv f_t - f_t^o.$$

where, f_t^I is knock-in put option and f_t is a plain vanilla put option and as we explained before, the option embedded in a RCN is just a barrier knock-in put option, f_t^I. Letting P(t, T) denote the present value of one dollar, the plain vanilla put options f_t at time t should be:

$$f_t = P(t,T)E_t\{e^{-\int_t^T \nu_s dS} \max(K - S_t, 0)\}$$

Letting g represent the probability density function of the average variance rate $\bar{\nu}$ in a risk-neutral world, and $Q = \{H < S_t\}$, then from equation (7) the value of a knock-out put option is:

$$f_t^o = \int_Q e^{-\int_t^T (r_s - d_s + \bar{\nu}_s)dS}(H - S_t)^+ g(\bar{\nu})d\bar{\nu}, \quad (8)$$

where, r is risk-free rate, d is dividend yield of a stock and $\bar{\nu}$ is the risk premium of the stock return. In our valuation model, g is the risk neutral join density function of Brownian and Lévy Jump innovation distributions. If we know the join characteristic function of this process, then we can derive the join density function from inverse Fourier transform technique. In fact, we have the join characteristic function[++]:

$$\Psi(.) = \Psi_B(.)\Psi_J(.). \qquad (9)$$

where, Ψ_B and Ψ_J are characteristic functions for Brownian motion and Lévy jump innovation, respectively. If the jump follows NIG distribution of the Lévy jump family, then g is the join density of Brownian motion and NIG distributon. Then Ψ_J should be the characteristic function of NIG distribution correspondent to its density function. The same is true for other distributions in the Lévy jump family, such as Variance Gamma (VG) distribution, etc. (see Appendix for details of the characteristic functions).

3.4: A DISCRETE TIME OF KNOCK-OUT PUT OPTIONS

In section 3.3 we described the stock option valuation model for a general term at a continue time. In this section, we simplify our model by considering the valuation in discrete time. We know that a knock out put option is a path - dependent option, which means the option value at time t is dependent on the value of previous time. For a knock-out put option, the option will be “knocked out” if it touches the barrier. So, a discrete time conditional knock-out put option, should have a payoff until t+Δt:

$$f^o(t+n\Delta t,0) = 1_{s(t+\Delta t)>H} x 1_{s(t+2\Delta t)>H} x \ldots 1_{s(t+(n-1)\Delta t)>H} x Max[H - S(t+n\Delta t),0]$$

where, S(t) is the underlying stock price at time t. At any time prior to the expiration of the RCN contract, if the stock price drops below a pre-specified barrier H, the put option is knocked out and becomes worthless. Analoging the analysis for equation (8) and (9), and considering the characteristic functions, we can rewrite the risk-neutral world knock-out put option value at time nΔt (with maturity T, T> nΔt) as:

$$f^o(t,n\Delta t,\phi) \equiv E^Q[H - S^\phi(t,n\Delta t)]$$

where, E is the expectation operator under conditional probability measure $Q \equiv \{H < S^\phi \mid t < n\Delta t\}$. ϕ is the argument of the characteristic function that define the probability density function. At time t we have:

$$E_t^Q[H - S^\phi(t,n\Delta t)] = E_t^Q[H] - E_t^Q[S^\phi(t,n\Delta t)] \qquad (10)$$

From (6), we have the second term in the equation (10) as

$$E_t^Q[S^\phi(t,n\Delta t)] = E_t[E_{t+1}[S_{n\Delta t}^\phi]] = S_t^\phi E_t[e^{\phi R_{t+1} + a(t+1,n\Delta t;\phi) + b(t+1,n\Delta t;\phi)h_{B,t+2} + c(t+1,n\Delta t;\phi)h_{J,t+2}}]$$

Using the property of iterated expectation, we have

$$E_t^Q[S^\phi(t,n\Delta t)] = S_t^\phi e^{a(t,n\Delta t;\phi) + b(t,n\Delta t;\phi)h_{B,t+1} + c(t,n\Delta t;\phi)h_{J,t+1}}$$

where,

$a(t,n\Delta t;\phi)$, $b(t,n\Delta t;\phi)$, and $c(t,n\Delta t;\phi)$ are coefficients. They are as[+++]:

$$a(t,n\Delta t;\phi) = \phi r_{t+1} + a(t+1,n\Delta t;\phi) + b(t+1,n\Delta t;\phi)\omega + c(t+1,n\Delta t;\phi)\omega k$$

$$-0.5\log(1 - 2b(t+1,n\Delta t;\phi)\,a' - 2c(t+1,n\Delta t;\phi)\,a' k)$$

$$b(t,n\Delta t;\phi) = -0.5\phi^2 + b(t+1,n\Delta t;\phi)(b' + a' c'^2) + b(t+1,n\Delta t;\phi)\,a' k c'^2$$

$$+\frac{(\phi-2b(t+1,n\Delta t;\phi)\,a'c'-2c(t+1,n\Delta t;\phi)\,a'kc')^2}{2(1-2b(t+1,n\Delta t;\phi)\,a'-2c(t+1,n\Delta t;\phi)\,a'k)}$$

$$c(t,n\Delta t;\phi)=b'c(t+1,n\Delta t;\phi)-\phi\xi(1)+\xi(\phi).$$

Now we can use the similar steps as in Bakshi and Madan (2000) to derive the knock-out put option valuation at time t as:

$$f_t^o(t,n\Delta t;H)=He^{-(r-d)n\Delta t}\Pi(t,n\Delta t)-S_t^{\phi}e^{a(t,n\Delta t;\phi)+b(t,n\Delta t;\phi)h_{B,t+1}+c(t,n\Delta t;\phi)h_{J,t+1}} \quad (11)$$

the characteristic function corresponding to the risk-neutral probability Π(t,nΔt) is,

$$\Psi=e^{(r-d)n\Delta t}\Psi(\bar{\phi}) \quad (12)$$

where, $\bar{\phi}=(\phi_1,...,\phi_{n-1,}\phi_n)$. The characteristic function in equation (12) can be derived from equation (9) for a join characteristic function. Then we can use the following property to estimate the probability Π(t,nΔt):

$$f(x)=\frac{F(x+h)-F(x-h)}{2h}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{Sin(ht)}{ht}e^{-itx}\psi(t)dt,$$

where, f(x) is a density function and F(x) is its probability function. ψ is join characteristic function of Brownian motion and Lévy jump innovation as we discussed above. The methodology we use above is based on n-dimensional Fourier inversion formula in Shephard (1991) and numerical method.

3.5 DATA

We obtained our RCNs data from Bloomberg where most of the issuers are global banks. RCNs are over-the-counter (OTC) products and market price data is very limited. We tested the RCN issues whose market prices were available. For the historical returns of the underlying stocks of RCNs, we also selected Bloomberg and we included longer time periods, including more than 4,000 of daily data for each stock ticker.

4. RESULTS

Before we discuss the results, we need to explain the estimation processes. There are mainly two important estimation processes. Firstly, we need to estimate all the parameters in order to forecast volatilities. We used a maximize likelihood method for filtrated affine GARCH (1,1) model. Since we used only the return shocks that were generated from Brownian increments for GARCH model, the shocks that were generated from the Lévy jump innovation must be filtrated out before we use the GARCH model. Then, we can combine the results with the Lévy jump volatilities to get the total volatilities. Secondly, from the results of the previous step, we can value the barrier option and then we can value the RCN. Within the second step, at each time period Δt, we check the option value to see whether it touches the barrier. If it touched the barrier, the option is worthless. Otherwise, the option has a positive value. Then we calculate the sum of the present values of each option premium. Now we put more effort to discuss the results. Because RCN data is limited, we test the model from many different perspectives. Firstly, we test whether the valuation results are closed to the market prices at a certain date. For example, we have the market prices for several RCNs as of May 21, 2011. We compare the valuation results from our valuation models to the market values at the same valuation date. Secondly, we compare the results with different models. Table-2 in the Appendix shows the results. The row of "Diff." refers to the differences of the absolute values from our model to the market quotes. We use the mid-values of bid and ask spreads as market quotes. From the table we can see that our valuations are very close to the market values for most of the RCNs except for the RCN with the underlying stock X -- the United States Steel Corporation. We believe that stock X has a relatively larger error than that of other stocks from the model because its option was deeply in-the-money, while others are at-the-money or out-the-money. In the table, we also list the estimated parameters from the three models: NIG, VG and pure GARCH. In each model, we valuated the RCNs with the same underlying stocks: IP (International Paper Company, BAC (Bank of America Corporation), X(United States Steel Corporation) and BBY(Best Buy Co. Inc). The results show that the model of VG overprices the RCNs and NIG closely prices the assets. This indicates that our model valuations are relatively close to market values. The result of VG overpricing is consistent with the research of others, such as Deng, et al (2012), even though our methodology is significantly different from their approach. In addition, we use our model pricing exotic options of the RCN other than European style option as they valuated. Both NIG and VG models show that the United States Steel Corporation has higher valuation errors compared to the GARCH only model. We believe that this phenomenon is due to the Lévy jump risks effects for a deep in-the-money exotic option. In Table-2, we also compared the valuation of four RCNs with four underlying stocks. For International Paper Company, the RCN is close to maturity and its volatility is relatively lower compared to the other stock options. As a result, its RCN value is almost the value of the note. This, in fact, is true and all three models show the same results because the options are zero. The parameters: gamma, alpha and beta are for the GARCH model, and the rest of the parameters are for the exotic option valuation model. All the parameters are higher for the companies where their options are in-the-money compared to the companies where their options are out-the-money with all three models. This is as we expected, because in-the-money options are more active or volatile than are the ones of out-the-money are. Our estimated parameter, μ, is zero for NIG model and is consistent with our expectation since μ does not play any role in the option valuation. The parameters of coefficient ξ and convexity adjustment η are not zero for most of the underlying stocks. This means an adjustment is necessary. For the results of pure GARCH model, the results show that the valuations are also close to market values but less impressive than the ones from the NIG model. For example, the pricing error from the NIG model for the RCN with underlying stock BAC is 0.03, while the pricing error from the GARCH model for the same underlying stock is -0.13. The overall results indicate that the NIG model can more accurately evaluate a RCN when its exotic options are not deeply in-the-money (ITM), while if its exotic options are deeply in-the-money, the pure GARCH model has better performance. However, since a RCN is a short-term security and during the life of its contract, its embedded exotic option is highly unlikely to be deeply ITM. Therefore, the NIG model should be a good approach for the valuation of RCNs. Overall VG model is the least performer among the three models.

5. CONCLUSIONS

In this paper, we developed a new asset pricing framework for one complex structured product -- RCNs. Our simulation results indicate that the valuation based on our new proposed model are close to the real market pricing, especially for those RCNs when their underlying stock options are not deeply in-the-money. In addition, our valuation methodologies are easy to implement and are tractable. We also tested several models in the Lévy jump family and our results show that the Normal Inverse Gaussian (NIG) model provides relatively better results than other members of the family thus leading us to believe that the NIG model is more suitable to evaluate complex structured products, such as RCNs. The research and methodology in this paper can be used in valuation of other equity-linked products.

ENDNOTES

*the distribution of $Y_{t_i+\Delta t} - Y_{t_i}$ is the same as Y_Δ for all t, $\Delta \geq 0$.

** See Bertoin (1996)

+For proof, see Ornthanlai (2011).

++see Shephard (1991) Theorem 2.

+++For proof, please see Ornthanlai (2011)

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APPENDIX

Table 1: Reverse Convertible Notes Trading at The Markets

Underlying Ticker	Coupon	Spot Price	Knock-In	Srike	Dividend	Maturity
IP US Equity	10.00%	31.31	75%	27.24	3.2548%	6/30/11
BAC US Equity	9.75%	11.58	75%	13.34	0.3404%	6/30/11
X US Equity	19.75%	44.96	75%	17.74	0.4380%	7/26/11
BBY US Equity	10.25%	31.33	70%	35.45	1.9665	7/29/11

Table 2: Parameter Estimations and Valuation Results of Different Models

	NIG Model				VG Model				GARCH			
	IP	BAC	X	BBY	IP	BAC	X	BBY	IP	BAC	X	BBY
MLE	-9633.846	-8648.725	-8547.6983	-8304.8	-866.857	-606.033	374.2886	-995.369	-9633.8459	-8649.0724	-8547.6983	-8304.7894
μ	0.0000	-0.0002	-0.0001	0.0004	0.0021	0.0025	0.0006	-0.0039	0.0000	-0.0002	-0.0001	0.0004
γ	0.0006	0.0009	0.0001	0.0011	0.0008	0.0663	0.0036	0.0159	0.0006	0.0008	0.0001	0.0011
α	0.0020	0.0083	0.0002	0.0017	0.0594	0.0796	0.0751	0.0328	0.0020	0.0076	0.0002	0.0017
β	0.0000	0.0361	0.8540	0.0000	0.7867	0.2748	0.8311	0.7879	0.0000	0.1301	0.8541	0.0000
c'	0.0004	0.0000	0.0000	0.2739	0.0799	0.8499	0.1250	0.0490	0.0001	0.0000	0.0000	0.0052
k	0.0000	0.0000	0.0000	0.0000	0.0835	0.3469	0.0927	0.1210	0.0000	0.0000	0.0000	0.0000
a'	0.0965	0.0000	0.5533	1.0000	0.1890	0.2411	0.1159	0.0614	0.0000	0.0393	0.6595	0.0077
b'	0.0943	0.0000	0.9534	0.2853	0.1428	0.0061	0.1073	0.0603	0.0000	0.0000	0.3918	0.0108
ξ	0.0000	0.0000	0.9974	0.0000	0.0628	0.0004	0.0230	0.0209	0.0000	0.0000	0.0000	0.0000
η	0.0185	0.9987	0.8932	0.1136	0.0002	0.0012	0.0257	0.0216	0.0100	0.0011	0.7665	0.0076
Option Value	0.00	1.37	16.30	3.38	0.00	0.89	12.03	1.06	0.0000	1.5216	14.7445	2.7511
RCN	101.12	99.73	85.19	98.53	101.12	100.21	89.46	100.85	101.12	99.57	86.74	99.16
Diff.	-0.88	0.03	-2.31	-0.62	-0.88	0.51	1.96	1.70	-0.88	-0.13	-0.76	0.01

Notes: The Diff. refers to the differences from the market values. IP is the stock ticker of International Paper Company, BAC is the stock ticker of Bank of America Corporation, X is the stock ticker of United States Steel Corporation and BBY is the stock ticker of Best Buy Co. Inc.

Chart 1

Chart -2

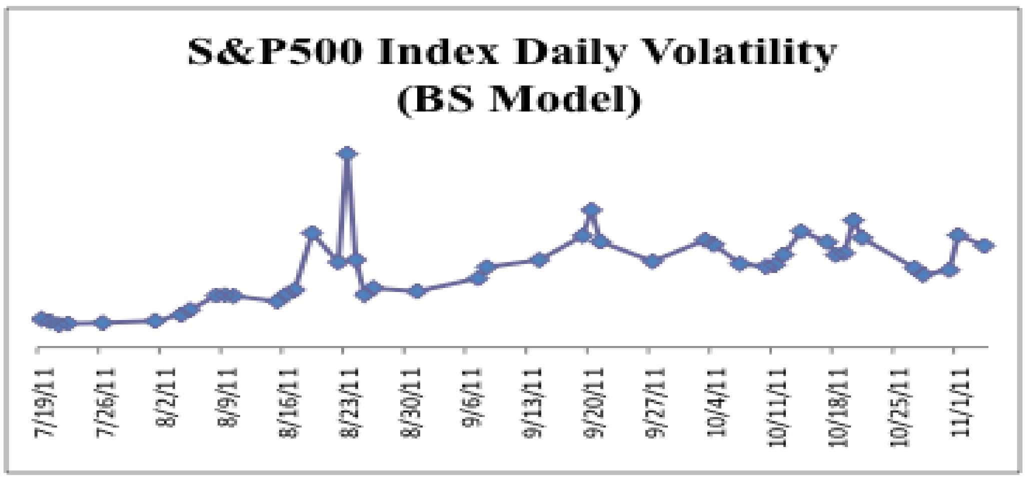
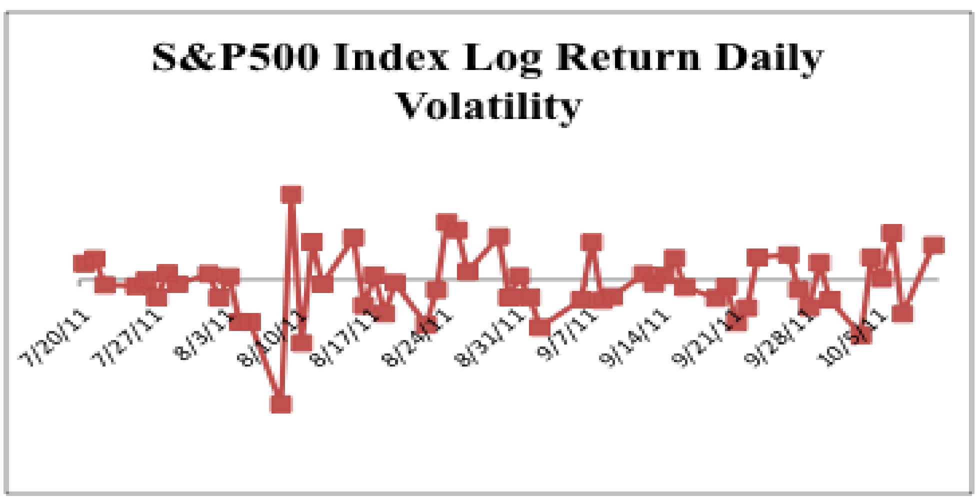


Chart-3



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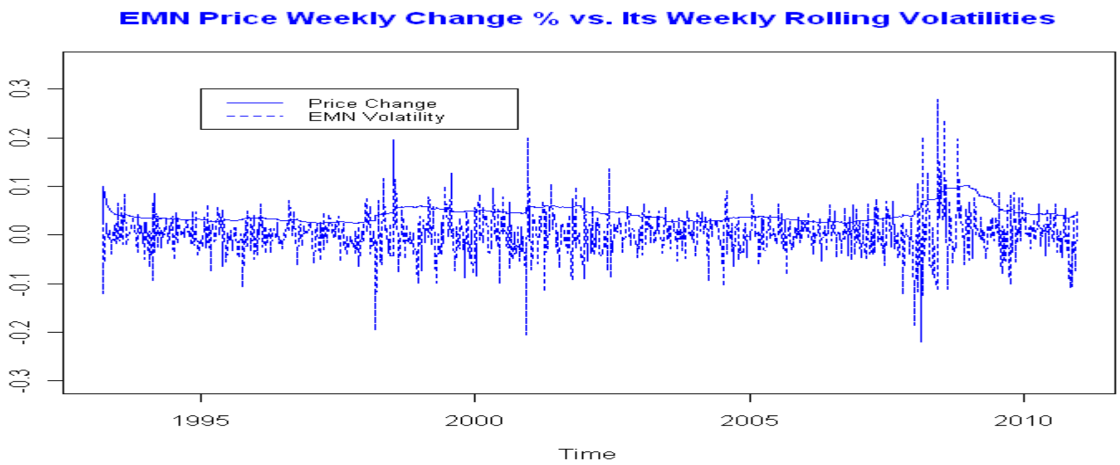

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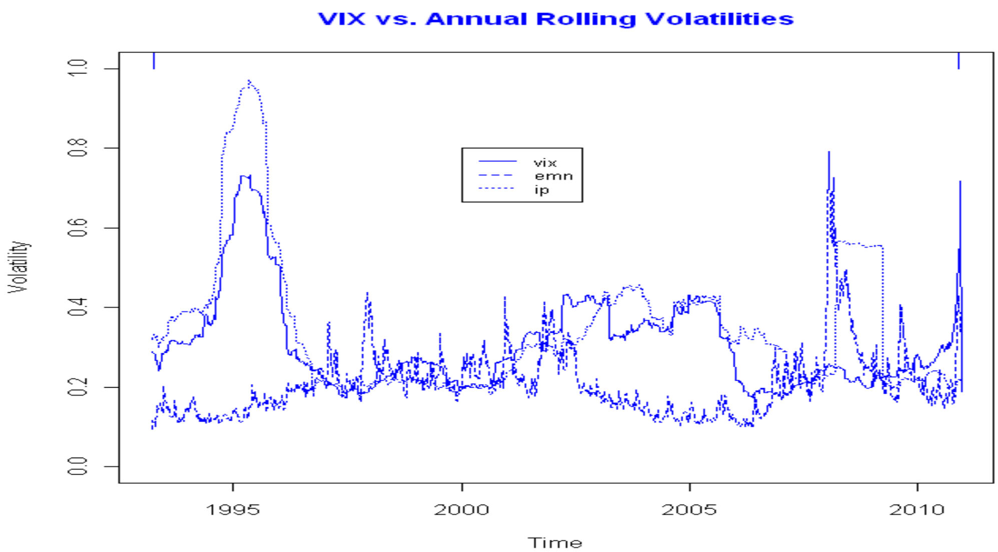

Chart-6