Pricing Weather-Based Irrigation Cost Insurance: Theory and Applications

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ABSTRACT

This paper uses weather insurance to price irrigation cost insurance contracts for agricultural crops. County-level yield data are used to estimate the probability distributions of irrigated and dryland wheat yields. A micro-economic model of production is developed that illustrates the relationship between rainfall, crop yields, costs of irrigation and profits. Alternative micro-econometric models are estimated to establish the effects of weather variables on wheat yield on dry land, and on the costs of irrigation. Weather-based insurance premiums are estimated using Monte Carlo simulations.

1. INTRODUCTION

The purpose of this paper is to examine how weather insurance products can be used to protect farmers against irrigation costs during drought years. Farmers who invest in irrigation are investing in an explicit form of self insurance to eliminate production risks. The cost of self insurance is a random variable jointly determined by rainfall, another random variable. Several studies have explored the issue of rainfall insurance in agriculture, ((Bardsley, Abey, and Davenport (1984), Hazell, Oram and Chaberli (2001), Gautman, Hazell, and Alderman (1994), Sakurai and Reardon (1997), and Turvey (2000, 2001)), but there are no known studies dealing with weather insurance to protect farmers against the increased costs of irrigation in drought years.

The paper proceeds with the theoretical framework of modeling the irrigation cost insurance and the specification of the yield-weather models. Empirical estimation of the irrigation cost-weather model and computation of the cost insurance premiums follow. Finally, conclusions to our research are considered.

2. ECONOMIC MODEL OF IRRIGATION COST INSURANCE

In this section, we develop an economic model of irrigation cost insurance to illustrate the relationship between a weather variable (rainfall =ω), crop yields y (ω), costs of irrigation c (ω) and profits $\pi(\omega)$. That is

$$Y_{max} = \mathrm{y}(\omega_{good})$$
(1)

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Since the maximum potential yield (y_{max}) acts as an absorbing barrier for all of the weather stated as good (ω_{good}), the marginal value product of irrigation above the threshold ω_{good} is zero. When rainfall falls below ω_{good}, the marginal productivity of rainfall increases at an increasing rate. Thus, the production function for output is

$$Y = \text{MIN}(y_{max}, \text{f}(\omega)) \tag{2}$$

where $y_{max} = f(\omega_{good}) > f(\omega)$. Therefore for $\omega < \omega_{good}$

$$\frac{\partial}{\partial \omega} \text{f}(\omega) > 0 \tag{3}$$

and

$$\frac{\partial^2}{\partial \omega^2} \text{f}(\omega) < 0 \tag{4}$$

which simply states that as water to the plants increases, plant growth increases but at a decreasing rate. We now consider the cost of irrigation C. The cost function is given by

$$C = \text{MAX}(0, \text{c}(\omega)) \tag{5}$$

If $\omega > \omega_{good}$ there is no need to irrigate so the cost is zero. Otherwise the cost increases as ω decreases. That is

$$\frac{\partial}{\partial \omega} \text{c}(\omega) < 0 \tag{6}$$

and

$$\frac{\partial^2}{\partial \omega^2} \text{c}(\omega) > 0 \tag{7}$$

The profit function can now be described in terms of the rainfall variable, output, and irrigation costs as

$$\pi = P\,\text{MIN}(y_{max}, \text{f}(\omega)) - \text{MAX}(0, \text{c}(\omega)) \tag{8}$$

where P is the price of the commodity. From (8), profits are given as Py_{max} if rainfall is adequate and $Pf(\omega)-c(\omega)$ if rainfall is inadequate. Furthermore, assuming that rainfall is inadequate, marginal profits obey

$$\pi_{\omega} = P\left(\frac{\partial}{\partial \omega} \text{y}(\omega)\right) - \left(\frac{\partial}{\partial \omega} \text{c}(\omega)\right) > 0 \tag{9}$$

Marginal profits are positive since the first term is increasing in ω, while the second term is decreasing in ω. In terms of risk and risk mitigation, the result states that as rainfall decreases, output will fall. In order to increase output, rainfall, in the form of costly irrigation, must be applied. Therefore in the years of drought, the dual effects of decreased yields and increased irrigation costs result in significant economic losses. Even if irrigation increases yields to its maximum level, the cost of irrigation remains as an uncertain cost to the producers. The essential economic elements to this problem from drought are the potential yield loss from lack of rainfall and the costs of mitigation. Since the latter is a risk reduction response to the former, then the insurable quantity is not necessarily yield per se, but the cost of irrigation. The yield loss component is economically significant only if irrigation is too costly or not available. Then, we can calculate the loss in profit (or the indemnity) as follows:

$$Z = P\,(y_{max} - y(\omega)) + c(\omega) \quad (10)$$

If irrigation is not available then $c(\omega)=0$, and the indemnity is only given by the yield shortfall. This is $P(y_{max} - y(\omega))$ and this is similar to conventional crop insurance. If irrigation is available then irrigation may increase yields so that the term $P(y_{max} - y(\omega)) \rightarrow 0$, but in this case $c(\omega)>0$ and this becomes the insurable event.

Since both $y(\omega)$ and $c(\omega)$ are functions of rainfall (a random variable), then yield and cost uncertainty can be established by defining the probability distribution functions for y and c. Let $g(\omega)$ be the probability distribution function for rainfall, then the indemnity function for profits is calculated by taking the expected deviation from the maximum potential yield. In the current discussion this has been denoted by the variable ω_{good} (11)

$$Indemnity = \int_0^{\omega_{good}} (P\,(y_{max} - y(\omega)) + c(\omega))\,g(\omega)\,d\omega$$

When irrigation is not available then the insurance form is similar to conventional crop insurance (CI) as

$$CI = \int_0^{\omega_{good}} P\,(y_{max} - y(\omega))\,g(\omega)\,d\omega \quad (12)$$

The final insurance product under consideration is irrigation insurance. Since y_{max} is an absorbing barrier for $\omega>\omega_{good}$ then a strategy that provides irrigation in the amount of ω_{good} - ω will have $y = y_{max}$ so that the first term in the general indemnity function (11) goes to zero leaving the irrigation cost recovery (ICR) indemnity function

$$ICR = \int_0^{\omega_{good}} c(\omega)\,g(\omega)\,d\omega \quad (13)$$

To estimate the indemnity schedules, we require information that is not readily available for underwriting purposes. Furthermore, yields, revenues or irrigation costs are not readily observable. Since rainfall is readily observable, a rainfall insurance policy can be designed to approximate the indemnities for crop, revenue, or irrigation cost insurance by using the following rainfall indemnity schedule (RIS);

$$RIS = z \int_0^{\omega_{good}} (\omega_{good} - \omega)\,g(\omega)\,d\omega \quad (14)$$

Z in equation (14) can be any value elected defined in the neighborhood of average cost of irrigation (AC):

$$z = AC = \frac{c(\omega)}{\omega} \quad (15)$$

By defining and empirically estimating *C=c (ω),* it is possible to map on this cost function the range of critical rainfall outcomes by defining the inverse function, $\omega = c^{-1}(C)$, and then derive an insurance premium.

3. CROP YIELD-WEATHER MODELS

3.1. Time-Series Yield-Weather Models

Wheat crop output is generally related through a production function to land, labor and capital. However, such a general function neglects the effects of weather variables on wheat yields. Based on the 1998 Kansas wheat performance test (Kansas State University, 1998), the critical weather factors are rain and heat. Therefore, we develop weather-crop models that link the year-to-year change in wheat yield (bu/acre) on dryland in year *t* (Δy_{dt}) to the year-to-year change in wheat yield (bu/acre) on irrigated land in year *t* (Δy_{it}), cumulative daily rainfall (inches) for the month *m* in year *t* (ΔR_{mt}) and cumulative degree-days above x degrees Fahrenheit for the month *m* in year *t* (ΔH_{mt}). Using this approach, we remove the effects of the trends in wheat yield and weather variables time-series data by calculating the first differences of the variables which illustrate the year-to-year changes. Our model and its variants are written as follows:

$$\Delta y_{dt} = \alpha_0 + \alpha_1 \Delta y_{it} + \sum_{m=1}^{4} \beta_m \Delta R_{mt} + \sum_{m=1}^{4} \gamma_m \Delta H_{mt} + \sum_{m=1}^{4} \delta_m \Delta(RH)_{mt} + e_t \quad (16)$$

where $\Delta y_{dt}=y_{dt}-y_{d,t-1}$, $\Delta y_{it}=y_{it}-y_{i,t-1}$, $\Delta R_{mt}=R_{mt}-R_{m,t-1}$, $\Delta H_{mt}=H_{mt}-H_{m,t-1}$ *and* $\Delta(RH)_{mt}=RH_{mt}-RH_{m,t-1}$. Using equation (16) the marginal responses of crop yields to year-to-year change in rainfall and heat are given by

$$\frac{\partial \Delta y_{dt}}{\partial \Delta R_{mt}} = \beta_m + \delta_m \Delta H_{mt} \quad (17)$$

and

$$\frac{\partial \Delta y_{dt}}{\partial \Delta H_{mt}} = \gamma_m + \delta_m \Delta R_{mt} \quad (18)$$

The effectiveness of specific-event weather insurance can be measured by the yield elasticity (δ_m) of rain and/or heat (expressions (17) and (18) must be positive).

3.2 Data Description

We used data on wheat crop yields from 1973-2001 for Ness County, Kansas (National Agricultural Statistics Service). Mean yields of wheat produced in irrigated land and dryland equal, respectively, to 45.75 bushels per acre (hereafter bu/acre) and 32.94 bu/acre. Irrigated wheat has the highest standard deviation (9.09 bu/acre) and the lowest coefficient of variation (19.87 bu/acre). Since the median values are greater than the mean values, wheat crop yields are found to be negatively skewed. Both yield distributions are also kurtotic.

We also used time-series data on daily precipitation and daily maximum temperature from 1951-2001 for the weather station of Ness County, Kansas (National Oceanic and Atmospheric Administration). The weather variables used, are cumulative daily precipitation in inches and cumulative degree-days (heat units) above 90 degrees Fahrenheit for critical months such as March, April, May, and June. The month of May shows the highest cumulative rainfall of 3.09 inches, followed by the month of June. The relative variability in rainfall is the highest during the month of March. The month of June is the hottest month with an average cumulative degree-day heat of 90 °F. The probability distributions of most weather variables are right-skewed and kurtotic.

There is a positive cross-correlation between irrigated wheat yield and dryland wheat yield (0.59). Wheat yields are positively correlated with rainfall in March and in April, and negatively correlated with rainfall in May and in June. Positive correlation coefficients in March and April indicate the importance of weather insurance for wheat farmers. Except for the month of June for dryland wheat yield, wheat yields exhibit a positive correlation with degree-days. The cross correlations between weather variables during the same month are negative.

3.3 Estimated Multiple Regression Yield-Weather Models

The base model of Equation 16 is referred to as model I. The Chow test was employed by restricting Model I through removal of interaction terms (Model II), heat (Model III), and rainfall (Model IV). In all cases, we failed to reject the restricted model. Table 1 presents the results of the four estimated crop-weather models. The R-squares of the models range from 0.47 to 0.75. Three models identify statistically significant effects of year-to-year changes in cumulative rainfall in the month of March and year-to-year change in irrigated wheat yield on year-to-year change in dryland wheat yields. By using the estimated parameters and the mean values of rainfall and heat for the month of March, we found that on the average, one inch increase in year-to-year change in rainfall during the month of March will increase year-to-year change in wheat yield on dry land by l.40 bu/acre.

Two models show statistically significant impacts of changes in heat during the month of June on changes in wheat yield. The positive effect of heat in June may be explained by the hot/dry conditions in

June that cause the crop to turn color and ripen much more rapidly (Kansas Agricultural Statistics Service, 2002).

Table 1: Estimated Regression Equations of Crop-Weather Models (1973-2001)

Variable	Model I		Model II		Model III		Model IV	
	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error
Intercept	-2.84	2.22	0.22	1.52	0.15	1.74	-0.33	1.56
Irrigated Wheat	0.27	0.18	0.31	0.17	0.37	0.15	0.46	0.15
March Rain	1.46	0.81	1.67	0.76	1.43	0.73		
April Rain	-0.91	1.37	-0.59	1.30	-0.09	1.44		
May Rain	0.48	1.03	-0.67	0.76	-0.86	0.79		
June Rain	1.11	0.91	1.01	0.84	-0.36	0.68		
March Heat	-6.84	4.35	-5.17	3.98			-4.88	3.03
April Heat	-0.22	0.18	-0.12	0.16			-0.19	0.16
May Heat	-0.03	0.13	-0.07	0.12			0.07	0.10
June Heat	0.08	0.04	0.06	0.03			0.02	0.02
March Rain*Heat	-0.53	0.71						
April Rain*Heat	0.06	0.07						
May Rain*Heat	0.07	0.06						
June Rain*Heat	-0.02	0.11						
Number of Observations	29.00		29.00		29.00		29	
F-Statistic	3.28		4.19		3.96		5.87	
RMSE	7.84		7.91		9.13		8.24	
R-Square	0.75		0.68		0.47		0.57	

4. IRRIGATION COST–RAINFALL REGRESSION MODELS

4.1 Cross-Sectional Cost-Rainfall Models

The effects of weather on irrigation cost are analyzed as follows:

(19) $$C = A\omega^{\beta}$$

where C represents the total or energy cost of irrigation (assuming a constant elasticity cost function), A is an intercept multiplier, ω, is annual rainfall, and β is the cost elasticity of rainfall. The two coefficients of the model A and β are expected to be positive and negative, respectively. The marginal cost of rainfall is given by

(20) $$\frac{\partial c(\omega)}{\partial \omega} = c'(\omega) = A\beta\omega^{\beta-1}$$

The necessary condition for rainfall insurance to be effective is that c' (ω) < 0 so that rain has an impact on the cost of irrigation. For the empirical estimation, the constant elasticity cost function is written equivalently as

(21) $$\ln C = \ln A + \beta \ln \omega$$

Based on the above cost function, the effectiveness of the policy is measured by the cost elasticity of rainfall,β.

4.2. Estimating Premiums Rates for Irrigation Cost Insurance

After estimating empirically equation (21), it is possible to map on the cost function the range of critical rainfall outcomes. Strike levels of rainfall are calculated by inverting equation (19). To determine the critical rainfall values, energy and total costs of irrigation are held constant at their means in the first case. The rainfall strike level is determined by $\omega^* = \omega(C^*, A, \beta)$ outcomes. The inverse function is defined as follows:

(22) $$\omega^* = \left(\frac{c(\omega)}{A}\right)^{\frac{1}{\beta}}$$

(23) $$\frac{\partial \omega^*}{\partial c} = \frac{1}{\beta}\left(\frac{c(\omega)}{A}\right)^{\frac{1-\beta}{\beta}}$$

The actuarially fair insurance premiums or the various costs to buy the options are computed as follows:

(24) $$premium = \frac{c(\omega)}{\omega} \int_0^{\omega^*} \left(\omega^* - \omega\right) g(\omega)\, d(\omega)$$

for a put-like rainfall insurance policy, and

(25) $$premium = \text{z} \int_0^{\omega^*} g(\omega)\, \mathrm{d}(\omega)$$

for lump sum payments where z is a constant dollar amount. The structure of the first weather derivative in this study is that of a European put option, where the option price is the cost of the derivative for the wheat farmer (i.e., the cost of purchasing rainfall insurance), and the strike is the rainfall threshold below which an indemnity is triggered. The put option would increase compensation at an increasing rate as the option moved further into-the-money. The option-like rainfall insurance product is triggered as soon as the rainfall measure becomes in the money. Once this event happens a fixed payout is made. The difference between (24) and (25) is that with the former, the indemnity increases with reduced rainfall, whereas in the latter a lump sum payment of z is paid if rainfall falls below $\omega*$ with a probability $\int_0^{\omega*} g(\omega)d(\omega)$.

4.3 Data and Model Specification

We used the 1998 Farm and Ranch Irrigation Survey (NASS/USDA) that provides cross-sectional data (48 U.S. states) on annual operating (maintenance and repairs, and energy) cost of irrigation. Unobserved heterogeneity is accounted for through the use of regional dummy variables. Average farm costs for machinery and repairs, energy and irrigation are $3,037.69, $6,157.75, and $9,195.44, respectively. The mean annual rainfall across all states is 39.17 inches. Computed correlations between rainfall and different categories of irrigation costs are negative. That is, a decrease in rainfall will most likely correspond with higher irrigation costs.

4.4 Estimated Irrigation Cost Models

We used the least-square dummy variable (LSDV) estimator to estimate the long-run cost function since the data includes regional differences in terms of climate. Equation (21) is modified and expressed as:

(26) $$\ln C_f = \ln A + \sum_{r=1}^{n-1} \alpha_r D_r + \beta \ln \omega_f + \varepsilon_f$$

where C_f is the total variable farm cost of irrigation for the state farm average; α_r is the regional-specific fixed-effect; D_r is the regional-effect dummy variable. Since the number of regions n is small, the estimation of equation (26) is achieved (using OLS) by keeping the constant term and adding n-1 dummies; ω_f is the vector of observed rainfall; β is the unknown cost elasticity parameter; and ε_f is the error term which is independently and identically distributed (i.i.d.) across average (state) farms and uncorrelated with the rainfall variable. The coefficient on rainfall, β, is expected to be negative. The regional fixed-effects represented by different dummy variables associated with α_r are expected to be positive or negative.

Table 2 presents the parameter estimates of the LSDV regressions of the energy cost model and total irrigation for the Midwest. Since Kansas belongs to the Midwest region, the Midwest regional dummy variable was dropped. These estimates with all dummy coefficients set to zero gives an estimate for this particular region. Thus, estimated models may be interpreted as long-run cost models of irrigation for the Midwest region. Both models have low R-Square but most of their coefficients are significant at least at the 0.01 level of significance. The parameters of the cost elasticity of rainfall are negative (an increase in rainfall will decrease the cost of irrigation). Energy cost of irrigation is more sensitive to change in rainfall than the total cost of irrigation. This is due to its negative correlation (-0.45) with the rainfall variable.

Table 2: Estimated Regression Equations of Cost of Irrigation (Cross Sectional Data)

Variable	Total Cost		Energy Cost	
	Estimate	Standard Deviation	Estimate	Standard Deviation
Intercept	8.93	1.83	8.79	1.94
Rainfall	-0.04	0.52	-0.12	0.55
New England	-1.66	0.50	-2.29	0.54
South	-0.25	0.45	-0.35	0.48
Mid Atlantic	-0.64	0.50	-0.96	0.53
Southwest	1.26	0.57	1.36	0.61
West	0.39	0.45	0.51	0.48
Number of Observations	48.00		48.00	
F-Statistic	5.54		8.53	
RMSE	0.90		0.95	
R-Square	0.44		0.55	

4.5. Results for Irrigation Cost Insurance

We used data on Ness County precipitation from 1951 to 2001 and computed the mean of cumulative rainfall which is 21.39 inches with a standard deviation 5.57 inches. Substituting 21.39 inches into the regressions resulted in an estimate of c (ω) of $4,548.03 for energy and $6,684.07 for total costs of irrigation. Using equation (15), th c (ω) from 0% to 25%, we extract the rainfall strike levels using equation (22).

To illustrate how irrigation insurance might work. Suppose that a farmer wanted to protect irrigation cost increases above the mean of $6,684. She can do so by purchasing a rainfall insurance contract that pays $300 for every inch of rain below 21 inches. For example, if rainfall is measured at 15 inches, the indemnity would be 6 inches*$300/inch = $1,800 to cover the cost of irrigation. If drought was severe and actual rainfall was only 5 inches, the indemnity would be 16 inches*$300=$4,800. Assuming a normal probability distribution function for rainfall, Monte Carlo simulations of equations (25) and (26) were used to compute insurance premiums. Table 3 shows the insurance costs when the insurance is tied to the energy costs of irrigation, while Table 4 reports the results for total irrigation cost. Two types of rainfall insurance products are used for illustration: the put option and the lump-sum payment option. Premiums for the put option are generated using equation (25). This will give the farmer the right to be compensated if the rainfall is below the strike level. For the lump sum option, the economic value of rainfall is assumed to be constant at the level of $2,000 and $1,000 for total cost and energy cost of irrigation, respectively. As shown in both tables, premiums are positively associated with strike levels of rainfall. To interpret these results consider the 10% increase row in Table 4. If an insured wants to protect or insure costs of about $5,002.83 then using equation (22) the corresponding level of rainfall to insure is 9.67 inches. Since, with a standard deviation in annual rainfall of only 5.5 inches per year, the cost of this insurance is low at only $0.90. For a lump sum payment of $1,000 if rainfall is below 9.67 inches, the insurance cost is $18. If, however, the farmer wanted to protect total costs in excess of the mean, then with a corresponding rainfall strike of 21.39 inches and an indemnity of $312.49/inch for each inch below 21.39 inches, the cost of insurance would be approximately $111.12. By accepting a deductible equivalent to a 5% increase in costs, the rainfall insurance falls considerably to a negligible $0.30/acre.

Table 3: Irrigation (Energy) Cost Recovery Indemnity for Ness County, Kansas

	Predicted Energy Cost ($)	Rainfall Strike Level (inches)	Premium Option Energy ($)	Premium Lump Sum Energy ($)
Mean	4,548.03	21.39	55.56	500.00
5% Increase	4,775.43	14.24	6.58	100.00
10% Increase	5,002.83	9.67	0.90	18.00
15% Increase	5,230.23	6.67	0.18	4.10
20% Increase	5,457.64	4.68	0.06	1.30
25% Increase	5,685.04	3.33	0.03	0.60

Table 4: Irrigation (Total) Cost Recovery Indemnity for Ness County, Kansas

	Predicted Total Cost ($)	Rainfall Strike level (inches)	Premium Option Total ($)	Premium Lump Sum Total ($)
Mean	6,684.07	21.39	111.12	1000.00
5% Increase	7,018.27	6.39	0.30	6.80
10% Increase	7,352.48	1.97	0.03	0.60
15% Increase	7,686.68	0.65	0.02	0.20
20% Increase	8,020.88	0.22	0.01	0.20
25% Increase	8,355.09	0.08	0.01	0.20

CONCLUSIONS

With a growing interest in weather-based insurance products, this paper has advanced the proposition that rainfall insurance can be used to insure against costly irrigation. It is intended to be illustrative and did not examine the efficacy of irrigation insurance relative to other forms of insurance such as crop insurance. The use of cross sectional models is far less desirable than using time-series costs for a particular farm, region or state. Nonetheless, this paper provides a reasonable starting point for examining how weather-based insurance product can be used to mitigate excessive irrigation costs for farmers.

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