A Peek into a Pandora's Box: Firm-Level Demand with Price Competition vs. Quantity Competition

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ABSTRACT

This paper explores the impact on firm-level demands under quantity competition tied to changes in the number of firms, product differentiation, and product group elasticity under price competition. It appears this deserves more attention. Some simple numerical analysis shows that inverse demand, compared with demand may be similar, more price elastic or less price elastic with increased competition beyond duopoly.   Indeed, it appears increased competition may lead to the competitive result with quantity competition while under identical market conditions the result is monopolistic competitive under price competition, unless market demand is highly inelastic. The numerical results reported here suggest that the economic landscape containing the interesting dilemma in which price and quantity competition yield significantly different firm-level demands and different equilibria, may very well need to be expanded from competition among the few to include competition among the many as well.

INTRODUCTION

This paper explores the impact increased competition may have on the distinction between demand and inverse demand faced by an individual firm. In the case of duopoly, inverse demand is less price elastic than demand regardless of the degree of product differentiation. (See Singh and Vives 1984). Vives (1999) and (1987) suggests why the distinction between price and quantity competition when the product is differentiated among many firms has not been addressed extensively yet.   If the residual price elasticity of demand faced by a firm is unbounded with increased competition, it leads to the competitive result with no distinction. If the residual price elasticity of demand remains bounded as the number of firms is increased, it leads to monopolistic competition, but cross-price elasticities will vanish as will any difference.   The distinction would remain in kind as *n* is increased, but only diminish in degree at most. Also, in the less technically limiting case which includes cross effects terms, inverting the demand relationship may itself be daunting with increased numbers, impeding any comparison at all.

Nevertheless, Hackner (2000) explores the distinction between price and quantity competition with the possibility of many sellers when product quality varies across sellers. With the aid of some restrictive assumptions, he shows that when there are more than two firms, quality varies greatly, duopoly results may be reversed, suggesting further exploration in this arena may be justified.

This paper develops a method for analytically inverting log-linear or linear demand systems involving any number of firms under any degree of product differentiation. Some simple numerical analysis then shows that inverse demand compared with demand may be similar; or more or less price elastic with

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increased competition beyond duopoly. Indeed, it appears increased competition may lead to the competitive result with quantity competition while under identical market conditions the result is monopolistic competitive under price competition, unless market demand is highly inelastic.

THE DEMAND SYSTEM

Demands faced by individual firms are assumed symmetrical. Log-linear demand for a representative firm under price competition is as follows.

(1) $\ln q_i = b \ln p_i + g_n \ln p_j + ... + g_n \ln p_n + \ln C$

Where:

p_i is firm i's price;

q_i is firm i's quantity demanded;

b is the firm's price elasticity of demand;

g_n is pair-wise cross-price elasticity and

C is a constant, and

$i \neq j$.

We assume $b < 0$, $g_n > 0$ for substitutes, and

(2) $b + g_n(n\text{-}1) = m$

Here m is the market demand elasticity for the good produced by the product group[1]. Equation (2) states the net response of buyers to a 1% change in firm i's price will equal the market demand elasticity for the product group. This implies $|b| \geq g_n(n-1)$. These assumptions ensure that (1) may be inverted[2].

Inverting (1) yields:

(3) $\ln p_i = \beta_n \ln q_i + \gamma_n \ln q_j + ... + \gamma_n \ln q_n + \ln c$

Where:

q_i is the quantity offered for sale by firm i;

p_i is the willingness to pay for q_i;

β_n is firm i's quantity elasticity of demand;

γ_n is pair-wise cross-quantity elasticity; and

c is a constant.

With n = 2,

(4) $\beta_2 = \dfrac{b}{\left(b^2 - g^2\right)} < 0$,

(5) $\gamma_2 = \dfrac{-g}{\left(b^2 - g^2\right)} < 0$, and

With the number of firms, *n*, greater than 2, the analytical inversion process can degenerate into unmanageable complexity.

ANALYTICAL INVERSION OF THE DEMAND SYSTEM WITH *n* >2

For internal consistency, (2) implies that these elasticities must be unaffected by a change in the number of firms in the product group. Otherwise, omissions or double counting must be occurring. It may be that these elasticities are in fact affected by a change in *n*, but the impact must be distributed on both sides of (2) to preserve the equality[3]. This property is general and should apply no matter what functional form is used to model the firm's demand. This means that the character of (2) is completely determined by the nature of the product and is independent of *n*. We can then define

(6) $g = g_n(n-1)$

We will refer to *g* as the product group cross-price elasticity. It is the percentage change in the demand of the firm's competitors, as a group, in response to a one percent change in the firm's price holding other prices constant. This measures total substitution possibilities inside the product group. It should move along with $|b|$ in response to changes in product differentiation among firms inside the product group. It is similar to $|b|$ in this regard, but unlike $|b|$, it does not include added substitution opportunities outside the product group.

Applying the same reasoning used in the case of firm *i*'s demand function,

(7) $\beta_n + \gamma_n(n-1) = \mu$

holds under any conditions, where μ is the inverse market demand elasticity for the product group. It follows that:

(8) $\mu = 1/m$, also under any conditions.

It will be useful to define a criterion for measuring the degree of product differentiation under quantity competition[4]. Set *n* = 2, divide both sides of (7) by μ:

(9) $\left(\dfrac{\beta_2}{\mu}\right)+\left(\dfrac{\gamma_2}{\mu}\right)=1$

This implies:

(10) $\left(\dfrac{\beta_2}{(\beta_2+\gamma_2)}\right)+\left(\dfrac{\gamma_2}{(\beta_2+\gamma_2)}\right)=1$

The first term of both (9) and (10) is the own quantity portion of μ and the second term is the cross quantity portion of μ. With a perfectly homogeneous product, an increase in production from any firm will impact buyer willingness to pay equally, and all of the quantity elasticities will be equal. The own and cross quantity portions of μ will be 0.5 in (9) and (10).

With product differentiation, the firm will have some ability to isolate itself from the impact of an increase in production by another firm in the product group. The own quantity portion of μ will therefore be greater than 0.5 and the cross quantity portion of μ will be less than 0.5 in (9) and (10) for the case of a differentiated product. The nature of the product, therefore will be reflected in the relative size of shares in (9) and (10) for the case of $n = 2$.

For a fixed and given product group quantity elasticity, μ, as n is increased, each of the left side terms in (7) must be reduced proportionally, asymptotically approaching 0 as n approaches infinity. In the case of homogeneous products, this would correspond to perfectly elastic firm demand under perfect quantity competition. With differentiated products, the process would always preserve relatively larger absolute values for own quantity elasticity as compared to any single pair-wise quantity cross elasticity as n is increased. With symmetrical demand across firms, the relationship, (10), between β_n, and γ_n, in this process of increasing n, is preserved in that:

$$(11) \quad \frac{\beta_n}{\left(\beta_n + \gamma_n\right)} + \frac{\gamma_n}{\left(\beta_n + \gamma_n\right)} = 1$$

for any pairing of firm i with one of the other n-1 firms. This is because with symmetrical demands, the impact of increasing n is distributed proportionally across firms. It follows that the own quantity portion of any two firms' share of μ when $n > 2$ will always be equal to the own quantity share of total μ when $n = 2$.

$$(12) \quad \left(\frac{\beta_n}{\left(\beta_n + \gamma_n\right)}\right) = \left(\frac{\beta_2}{\mu}\right)$$

This will be true despite the fact that:

$$\beta_n < \beta_2 \quad \text{and} \quad \beta_n + \gamma_n < \mu$$

This character of the firm's inverse demand will be completely determined by the nature of the product and the degree of product differentiation. Therefore, it will be independent of the number of firms. In addition, from (7) we know that all of the quantity elasticities, own and cross, must sum to equal the product group quantity elasticity, μ. Equations (12) and (7) provide two conditions which must hold for each firm's demand function under quantity competition with n firms in the product group. To solve (12) and (7) for β_n and γ_n, as a functions of the duopoly solution, first solve (12) for γ_n:

$$(13) \quad \gamma_n = \beta_n \left(\frac{\mu}{\beta_2} - 1\right)$$

Substitute (13) into (12), and factor out β_n:

$$(14) \quad \beta_n = \frac{\mu}{\left(1 + (n-1)\left(\left(\frac{\mu}{\beta_2}\right) - 1\right)\right)}$$

With β_n from (14) γ_n may be found using (13). Equations (14) and (13) are the solution to (12) and (7). These relationships are perfectly general and should hold for any inverse demand system regardless of functional form.

The conversion of b to β_n expressed entirely in terms of the $n = 2$ case under price competition may be derived from (14) by substituting in (4) and (5) from the two firm inversion. This is because in this particular functional form, (4) and (5) express inverse demand elasticities in terms of the corresponding demand elasticities. Some algebra yields:

(15) $$\beta_n = \frac{1}{\left(1-(n-1)\dfrac{g}{b}\right)m}$$

In addition, given β_n, from (15), γ_n may be most easily determined by using (13) and substituting in (4) and (5) from the 2-firm inversion:

(16) $$\gamma_n = \beta_n\left(\frac{-g}{b}\right)$$

So any firm-level log-linear demand function with price competition among n firms, (1), may be converted into its corresponding inverse firm demand function with quantity competition among n firms, (1), preserving the appropriate relationship between firms and the nature of product as given by elasticities of the price competition demand function[5].

COMPARING DEMAND AND INVEARSE DEMAND WITH THE NUMBER OF FIRMS VARIABLE

Equations (15)-(16) can show how a change in the number of firms, product differentiation, and product group elasticity under price competition will translate into impacts on the corresponding quantity competition firm-level demand relationship.

Own Price Elasticity and the Number of Product Group Firms

Figure 1 compares price elasticity of demand under price competition and quantity competition when the product is differentiated. In order to do this $|1/\beta_n|$ using (15) is plotted along with $|b|$. In this example, $|b|$ is set at 11.5, g is set at 10, and therefore, $|m| = 1.5$.

Here the residual price elasticity of demand remains perfectly bounded under price competition as the number of firms is increased, but appears to be completely unbounded under quantity competition as the number of firms is increased, other things equal. As a result, with very small numbers, inverse demand is more price inelastic than demand. Beyond 9 firms in this example, $|1/\beta_n| > |b|$, and as n increases further we would expect to find the competitive result with a sufficiently large but finite n. Under price competition, with identical market conditions the same process yields the monopolistic competitive result.
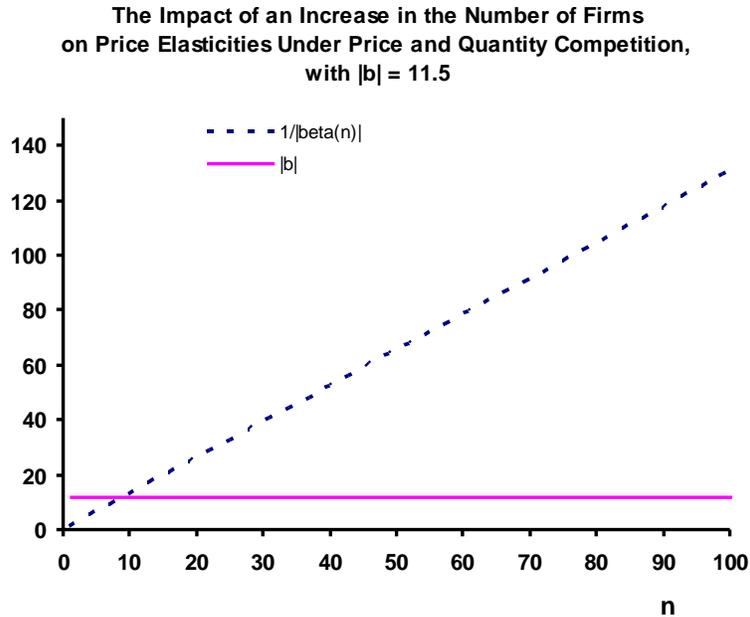
**The Impact of an Increase in the Number of Firms
on Price Elasticities Under Price and Quantity Competition,
with |b| = 11.5**

Figure 1

Own Price Elasticities and Changes in Product Differentiation within the Product Group

Changes in b rooted in changes in m is shared by all firms. Because of this, product differentiation distinguishing a firm's goods from other firms' in the product group under price competition will be defined for this study as changes in both b, and g, that preserve m unchanged[6]. Greater product differentiation measured this way has a significantly smaller impact on (15) as compared to the impact of a change in n. An increase in product differentiation will reduce both g and b. This will reduce the ratio ($-g/b$) in (15), but only slightly, unless both $-g$ and b are relatively small. These conjectures are illustrated in Figure 2.

Again, results from equation (15) are inverted so that we can compare b with $1/\beta_n$. In this example m is held at -1.5. The elasticity calculations in Figure 2 are in absolute values. As expected, $|b|$ increases linearly as the degree of product differentiation is reduced. Vives (1999), p. 168, asserts that demand under price and quantity competition will be identical in the limit of monopolistic competition because cross effects will be absent there, or $b = 1/\beta_n$ in the notation of this paper. The results reported here are not at odds with this. They do, however, show that Vives conjecture holds only in the extreme limiting case. This situation is shown in Figure 2 as the point in the lower right corner where $|b|$ has been reduced to 1.5. Holding m constant, (2) implies at this point $g = 0$. Here $b = 1/\beta_n$. As the degree of product

differentiation is relaxed by the smallest possible degree, however, Figure 2 shows $|1/\beta_n|$ will "snap" away from $|b|$. It appears that even a small presence of cross effects in the firms demand relationship can translate into dramatic increase in $|1/\beta_n|$ in this process. This is why the expected asymptotic approach toward identical results for demand and inverse demands as n is increased from 2 does not happen. After the "snap", $|1/\beta_n|$ remains unresponsive to further reductions in product differentiation.
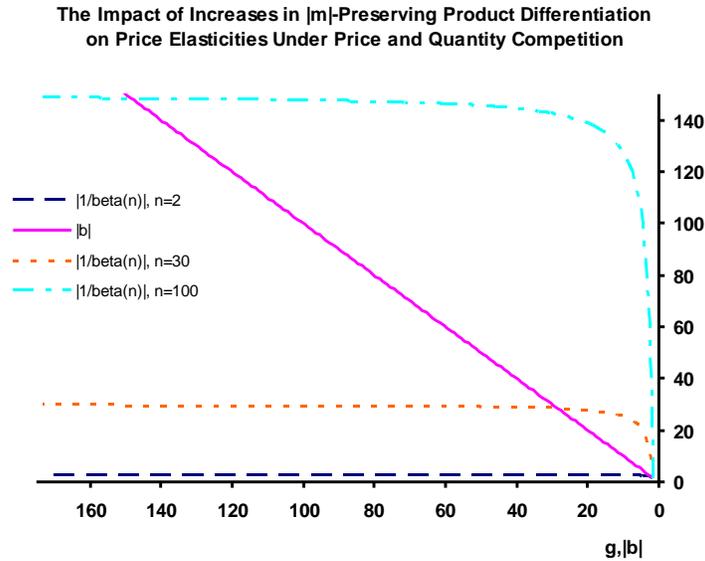
**The Impact of Increases in |m|-Preserving Product Differentiation
on Price Elasticities Under Price and Quantity Competition**

Figure 2

Own Price Elasticities and the Impact of Changes in Product Group Price Elasticity

A reduction in m, while preserving the degree of product differentiation may be accomplished by increasing g, while holding b and n constant in equation (15). Equation (15) suggests the impact of such an adjustment on β_n is ambiguous. This is illustrated in Figure 3 for $1/\beta_n$. In Figure 3, $|b|$ is set at 11.5, and g varies from 0 to 11.5, moving left to right. This implies $|m|$, will vary from 11.5 down to 0. With $n = 2$, $|1/\beta_n| < |b|$, with this process as expected. With larger n, however, it appears that $|1/\beta_n|$ may initially increase rather than decrease, so that it is greater than $|b|$ as $|m|$ is reduced modestly. Figure 3 suggests that this effect could be mitigated and may reverse if $|m|$ is reduced more dramatically. Indeed, as $|m|$ approaches perfectly inelastic, $|1/\beta_n|$ approaches perfectly inelastic as well, while $|b|$ remains unchanged! This means all of the results summarized with Figures 1 and 2 can be reversed with sufficiently inelastic market demand.
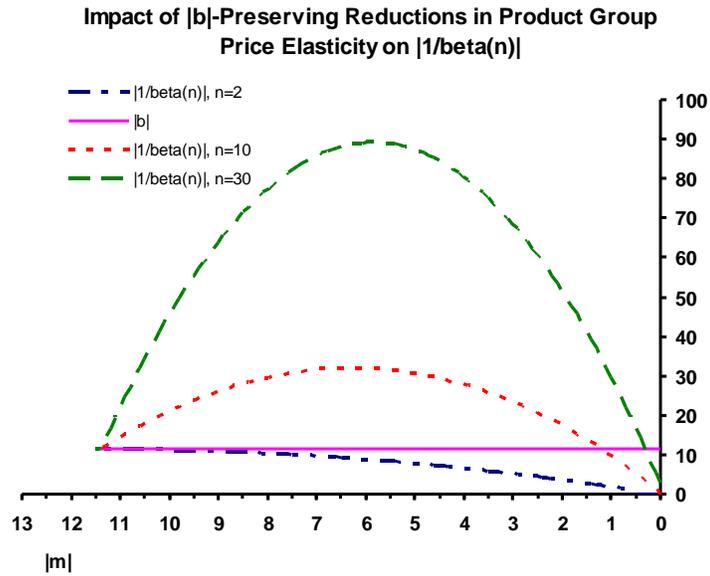
**Impact of |b|-Preserving Reductions in Product Group
Price Elasticity on |1/beta(n)|**

Figure 3

SUMMARY AND CONCLUSIONS

This paper explores the impact on firm-level demands under quantity competition tied to changes in the number of firms, product differentiation, and product group elasticity under price competition. It appears this deserves more attention. Some simple numerical analysis shows that inverse demand, compared with demand may be similar, more price elastic or less price elastic with increased competition beyond duopoly. Indeed, it appears increased competition may lead to the competitive result with quantity competition while under identical market conditions the result is monopolistic competitive under price competition, unless market demand is highly inelastic. The numerical results reported here suggest that the economic landscape containing the interesting dilemma in which price and quantity competition yield significantly different firm-level demands and different equilibria, may very well need to be expanded from competition among the few to include competition among the many as well.

ENDNOTES

1. Removing logs from both sides of (1), and (3) yields the linear demand system. When this is done, b, g, g_n, and m on the one hand, and β_n, γ_n, and μ, on the other are coefficients. All of the logic and method discussed in the log-linear example of the text transfers directly and generalizes to the linear case with elasticiites replaced with coefficients. This is not done in the text for space considerations.
2. See Cheng (1985).

3. Any impact on (1) or (3) tied to a change in n should therefore be restricted to an impact on the constant in each case, since impacts on elasticities would cancel. The specific form of the impact is an interesting issue, not explored here.

4. For consistent notation, β_n will be used to represent own quantity elasticity when $n > 2$. β_2 will represent own quantity elasticity when $n = 2$. Similarly, γ_n will represent pair-wise cross elasticity when $n > 2$. γ_2 will represent pair-wise cross elasticity when $n = 2$.

5. This would require additional information on the particular impact a change in n would have on C, and c, so that the constant may be inverted as well. These issues have no influence on this study and are not explored here.

6. This should show up under quantity competition as a redistribution of own vs. cross shares in (11).

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