ON "ARBITRAGE" AND MARKET EFFICIENCY: AN EXAMINATION OF NFL WAGERING

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ABSTRACT:

For several decades researchers have searched for possible inefficiencies in sports gambling markets. Most profitable strategies have failed to produce profits over the long run. The one consistently profitable strategy that has been studied extensively involves taking advantage of differences between contracts offered in different regional gambling markets. The main purpose of this paper is to explain why and how these differences affect the profitability of wagers. I describe two ways that one could take advantage of these different contracts, loosely defining one of these opportunities as "arbitrage". This paper examines the circumstances under which inter-market gambles in the NFL can be made with an expected profit. Of course, it is expected that as with all arbitrage opportunities, such circumstances are expected to be rare and short-lived, except when betting against a local team.

Pankoff (1968) was the first to test the efficient markets hypothesis for sports gambling markets, and found little evidence of inefficiency in NFL markets. A search for strategies that can "beat the market" ensued, and though researchers appear to report conflicting results, a pattern of regularity has emerged over time. Most findings of inefficiency appear to fall in one of three categories:

1) Testing many strategies and producing possible type I errors
2) Risk neutral strategies against risk averse (or risk loving) bettors
3) Taking advantage of different odds or lines offered by different bookmakers

Over time, the only profitable strategies proposed in the literature that have been shown to remain profitable in out of sample tests fall into category 3, where a bettor finds an "advantage". Such advantages involve choosing the best odds or lines offered from among several bookmakers. These advantages were found to be profitable in papers by Vergin and Scriabin (1978), Tryfos et al. (1984), and Badarinathi and Kochman (1996a), as well as other authors in the economics literature. However, no previous research has examined these

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advantages in detail, explaining the theory behind how these advantages result in profitable opportunities. This paper explains the theoretical and empirical underpinnings of such advantages, and proposes a method for exploiting them if they can be found. We construct pairs of bets that can loosely be termed *arbitrage*: buying and selling contracts in different markets that have either an expected (when betting using lines) or guaranteed (when betting in odds) profit.

We begin with a brief discussion of several examples of research in each of the above categories. In Section 2 we will describe why different bookmakers often offer different lines or odds. Section 3 briefly introduces vocabulary and explains the structure of the NFL wagering market. Section 4 will suggest a method for taking advantage of different lines offered by bookmakers by making opposite bets in two markets. Section 5 examines recent data from the NFL and finds the necessary conditions for profitable wagers. Sections 6 and 7 discuss and extend the results. Section 8 derives the necessary conditions for profitable wagers in odds between two markets, followed by the Conclusion (Section 9).

1. Previous work on betting

While in 1998 $2.3 billion was spent on *legal* sports gambling in Nevada, it is estimated that between $80 and $380 billion was spent gambling on sporting events illegally (Macy 1999).[1] The sheer size of the illegal market relative to the legal one makes it clear that focusing on the legal market alone fails to capture the complete picture. However, as evidenced by the huge range of estimates of the size of the illegal market, little can be said with certainty. We are severely limited in our ability to study these illegal markets, so most authors focus on the "Vegas line". Many authors have tried to uncover inefficiencies in Vegas lines for NFL games and construct betting strategies that have a positive expected return.

Type I Errors?

Many studies test historical data using a large number of strategies. Often without a guiding theoretical basis, the data are searched for possible profitable wagers that *would have won* more than $\frac{11}{21}$ of the time. The tested strategies range from the simple [bet on the underdog (Vergin and Scriabin 1978), bet on (against) teams that consistently beat (lose to) the point spread (Sturgeon 1974)] to the complex [using regressions to predict winners (Zuber et al. 1985, Osborne 2001)].

Most findings of inefficiencies in markets have been found to be sample specific, implying that the researchers may have committed type I errors. Researchers should always be mindful of this possibility for error, although discussion of type I errors is extremely rare in this literature. In studying gambling market efficiency, setting an alpha = .05 means that for every 20 strategies tested, we expect that one will appear profitable even when the market is truly efficient. Of course when many hypotheses are tested, more type I errors will likely occur.

For example, after testing 15 hypotheses, Gandar et al. (1988) find one profitable strategy with alpha = .05. Woodland and Woodland (2000) test 48 hypotheses, and reject 7 at alpha = .10. The numbers of profitable strategies found are similar to (but slightly greater than) what one would expect to find when, in fact, no profitable strategies exist. Little can be done to ameliorate this problem in practice[2], however authors should use caution when interpreting results. For example, when Badarinathi and Kochman (1996b) test 116 hypotheses, and reject 7 at alpha = .05, they correctly conclude that the findings of inefficiency are likely to be spurious.

Risk Preference

Several authors, most notably Woodland and Woodland (1991) have examined the effects of risk aversion on bettors. Bettors are often risk loving, evidenced by making wagers with negative expected values, and betting on teams with which they have an emotional connection. Consistent with this proposition, several authors have found that racetrack bettors overbet longshots (e.g. Asch, Malkiel, and Quandt 1982). However, Woodland and Woodland (1994) find weak support for the opposite case in baseball.

When focusing on wagering in football, risk aversion plays little part because when betting using point spreads, each wager has approximately even odds of winning. On the other hand, risk loving behavior actually drives the main premise of this paper. Because bettors tend to bet heavily on the local favorite, regional differences between lines may appear. Sometimes these differences can give a bettor an advantage.

Finding an Advantage: Differing Lines

Picking a strategy (such as betting the underdog) **AND** obtaining an “advantage” in a local bookie’s line relative to the Vegas line is one tactic that has appeared profitable in many studies over many samples (e.g. Vergin and Scriabin 1978, Tryfos et al. 1984, Badarinathi and Kochman 1996a). For example if a betting rule suggests that you should bet on Team A, and the Vegas line indicates that Team A is favored by 5 points, only bet on Team A if you can find a line of 4, 3.5, or 3 in another market. This is referred to as finding an advantage of 1, 1.5, or 2 points, respectively. Vergin and Scriabin, Tryfos, and Badarinathi and Kochman obtained winning percentages of 60.59, 59.26, and 56.03 in three different samples when betting on underdogs, assuming one could obtain a two-point advantage. As will be shown, the essential element is not the strategy itself, but the size of the advantage obtained. The key element is finding different lines or odds offered in different markets.

2. Different odds for the same game

One large factor that differentiates gambling from other financial markets is that gambling on sporting events is illegal almost everywhere in the U.S. This fact likely causes inefficiencies in the market due to separation of local, illegal gambling markets and increased information costs. Vergin and Scriabin(1978) discuss the importance of getting the “best” odds or lines possible when placing bets.

They note that the point spreads often change two to three points after they are initially published in Las Vegas, and usually vary across cities. Regardless of your strategy, it is important to make sure that you select the most favorable line or odds, increasing your expected profits. Rosecrance(1988) describes a typical betting day at Lake Tahoe:

> Bettors frequently get together ... to compare numbers (odds or points offered by the various sports books) and to discuss wagering opportunities with other regulars. Some call acquaintances in Las Vegas or Reno to check the numbers being offered in those locations.

An example of such a difference occurring was the 1969 Super Bowl, where the Colts were favored over the Jets by 20 in Baltimore, but only by 17 in New York (Merchant 1973, p. 41). The fact that different terms are offered on the same event brings up an interesting question: since spreads differ across space, and also change over the time before a game starts, what are the conditions necessary to find profitable opportunities to bet across markets?

3. A primer for the risk averse

In order to be concrete, we now give an introduction to this market and define some terms. A bookmaker, or **bookie** is one who accepts bets from individuals. These individuals normally serve a limited geographic area because of legal restrictions. When a bookie accepts a bet, it is a contract as follows:

> **The bookie sells the buyer n units of a contingent claim such that if team A scores more than *x* more points than team B, the buyer wins $n. If team A scores less than *x* more points than team B, he loses $(1+v)n. If team A scores exactly *x* more points than team B, the purchaser does not win or lose any money** (a *push*).

Let's call the type of bet just described a **bet on team A**.

The variable ***x*** is often called the **line** or the **spread**. A favored team must win by more than ***x*** points to win a bet on that team, and conversely an underdog must lose by fewer than ***x*** points (or win outright) in order to win a bet on an underdog. Thus let ***x*** be positive if A is favored to win, and negative if A is the underdog. Local bookies will increase or decrease the line to try to keep the amount bet equal on each side, with the goal of taking no personal risk.[3] However, when a bet is made, it is "locked in" at the ***x*** given at that time. The variable *x* can either be an integer, in which case pushes are possible, or non-integer (e.g. 3½) which makes ties impossible. If too many bets on team A are being bought, the bookie can make bets against team A look more attractive by increasing the amount ***x*** (i.e., by increasing the probability that a bet on team B will win). Bookies normally try to act as a broker between bettors, and profit only from their commission, ***v***.

The variable ***v*** is called a **vigorish**(vig). This is a percentage (usually 10 percent) added to all losing bets as a commission.[4] This is where the bookie makes most of his income, and for now we will assume that this is his only income. Of course, the bookie can strategically choose to bet against his customers, as opposed to simply acting as a broker. Strategic actions by bookies will not be considered in this paper.

4. On arbitrage opportunities

Because most bookies serve only a small geographical area, regional demand differences (and lack of information) may generate different spreads in different markets. Whenever a wedge in prices between markets occurs, there may be the opportunity to bet on a team to win a bet in one market, and lose in another.

Though these transactions do not obviously lend themselves to being called arbitrage (because profit is not *certain*), they come very close to meeting the definition supplied in *The MIT Dictionary of Modern Economics* (Pearce 1986):

> **arbitrage.** *An operation involving the simultaneous purchase and sale of an asset (e.g. a commodity or currency) in two or more markets between which there are price differences or discrepancies. The arbitrageur aims to profit from the price difference; the effect of his action is to lessen or eliminate it.*

Some readers may quibble with whether or not the transactions described in this paper constitute arbitrage; I implore the reader to forgive my loose usage of the word.

A simple example will make arbitrage in these markets more clear. Suppose that Miami is better than Denver, and everyone believes it (even the fans of Denver). Also suppose that the fans of each team demand bets on their team more than other consumers. In equilibrium, suppose that the line in Miami is 10 points and in Denver it is 5 points. If we buy n units of “bet on Miami” in Denver and n units of “bet on Denver” in Miami, we will see one of the following five outcomes *ex post:*

I. In region I to the left of zero, Miami loses the game. Just to the right of zero, Miami wins, but just barely. So, we will win our bet on Denver and lose our bet on Miami. Our payoff is (n - n(1+v)) = -vn.

II. In region II we win both bets. Miami wins by more than 5, and Denver loses by less than 10, winning both bets. Our payoff is (2n).

III. Region III is similar to region I, winning our bet on Miami, and losing our bet on Denver. Our payoff is -vn.

IV & V. The score could also be **exactly** a 5 or 10 point win for Miami. In the case of the 5 point differential, we win our bet on Denver and have a “push” on Miami. So, in these cases we simply win n.

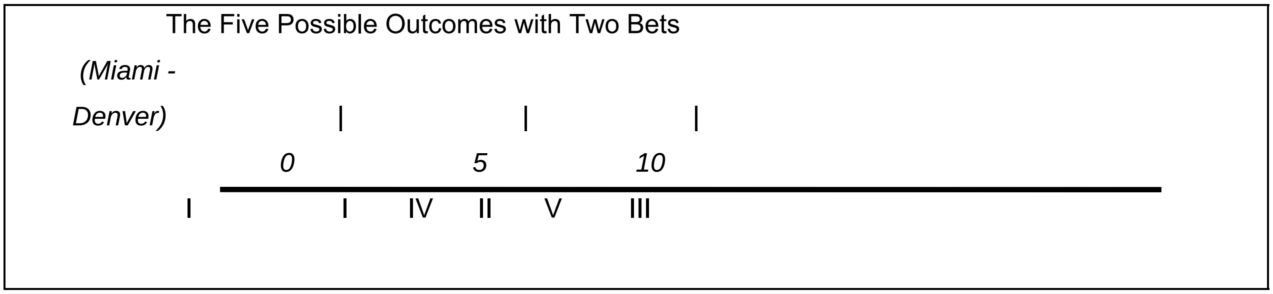

Figure 1: Five possible outcomes from an arbitrage using different point spreads of Miami favored by 10 (*in Miami*) and Miami favored by 5 points (*in Denver*). In regions **I** and **III**, we win one bet and lose one bet. In region **II**, both bets are won. If the outcome is that Miami wins by exactly 5 or 10 points, we win one bet, and neither win nor lose the other.

In order to make money in this above scenario, the probability that the score is in the arbitrage region or equal to one of its boundaries must be p* and $\bar{p}$ such that:

$$\bar{p}(n) + p^*(2n) > (1 - p^* - \bar{p})vn \qquad (1)$$

where $\bar{p}$ is the probability that the difference in scores equals one of the lines, and p* is the probability that the difference falls within the two lines. If we assume for just a moment that the points are distributed continuously, so that $\bar{p}$ = 0, then

$$p^* > \frac{v}{2+v} \qquad (2)$$

If the vig is the customary 10 percent, then p*, the probability we win both bets, must be at least 4.76 percent. Of course, these unrealistic simplifying assumptions are only for illustrative purposes; a more realistic derivation follows in section **5.**

When Pankoff(1968) analyzed the distribution of actual NFL game outcomes around the Vegas line using 1956-1965 data, he found a standard deviation of 15.58. Stern(1991) analyzed the accuracy of point spreads using NFL data from 1981, 1982, and 1984. He found that distribution of the difference between actual game outcomes and point spreads was not significantly different from a normal distribution with mean zero and a standard deviation of 13.86. Using data from 1997 and 1998[5] (473 games) and Las Vegas lines, we found that the standard deviation was 12.91 (Figure 2), and the data are still approximately normal with mean zero.[6] It is interesting to note that the Las Vegas lines seem to be more accurate in more recent data, less varied from the actual game outcomes. A more detailed exploration and explanation of this apparent trend would be an interesting addition to the literature. Are lines more accurate because of more efficient markets, or are the average score differences becoming lower, making them easier to predict?

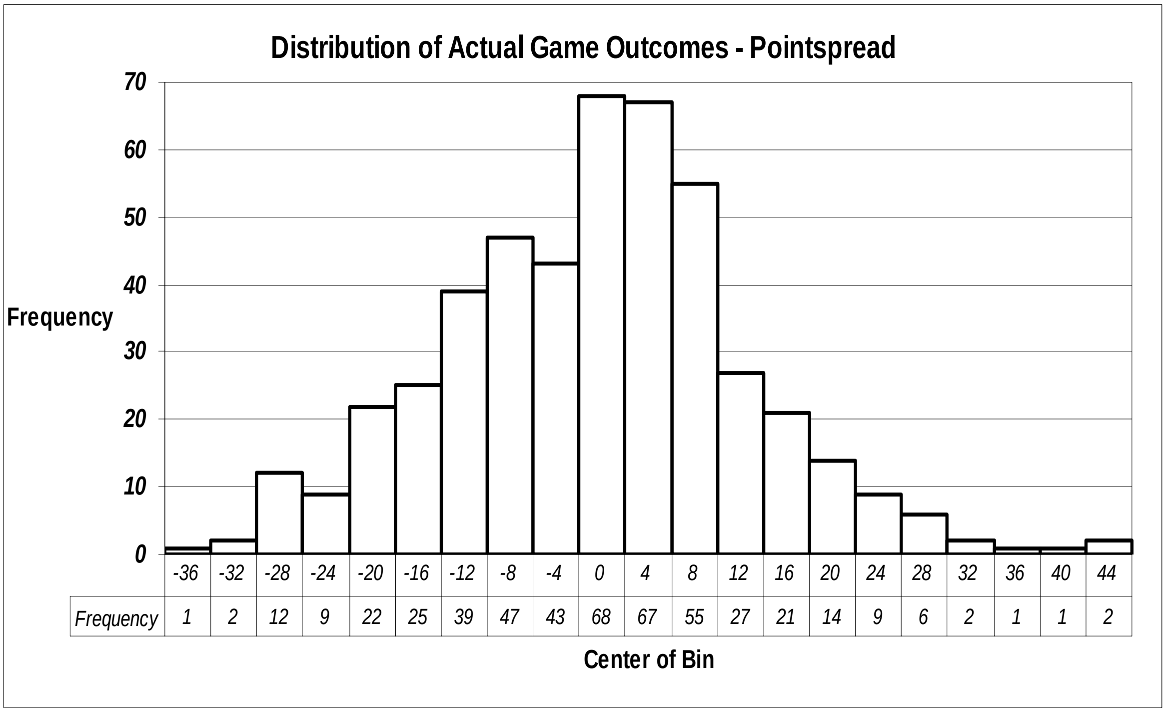

Figure 2: Histogram showing the distribution of game outcomes relative to the spread (Actual Game Point Differences - Las Vegas Line) for 1997 and 1998 seasons. The sample mean was -.23, standard deviation 12.91. A goodness of fit χ^2 test could not reject the assumption that the data come from a normal distribution with mean 0, standard deviation 12.91.

The unbiasedness of the Las Vegas lines as seen from the zero mean is also as expected, suggesting that the Vegas line is an unbiased estimate of true game outcomes. However, as the variance decreases, the difference in observed lines required to generate a sufficient p* decreases. Continuing with the simplification in equation (2), using Stern's estimate would require a minimum difference in the lines of 1.65 points to break even, whereas the more recent estimate requires only 1.54. Because we are constrained by ½ point intervals, this means that a with a **two point** difference between markets, one could expect to make money. This agrees with the research discussed in section **2** that found that a two point "advantage" could lead to a profitable wager.

If we suppose that an unbiased point spread is announced, and observe the line move up by one point in one market, and down by one point in another, then we would have a positive expected payoff from arbitrage. However, the above analysis is an unjust simplification derived in order to introduce the reader to the issues. Because ties and ½ point lines sometimes affect the probabilities substantially, we now offer a more accurate solution to our arbitrage problem.

5. A more formal treatment

Suppose that there is some true discrete distribution of ***x***, ***f(x)***. Let ***X*** be the expected value for a given game, which bookies know. Suppose that bookies are acting merely as brokers, and market forces drive a wedge between the lines in two local markets to x_A and x_B, with $x_A < x_B$. Then the expected profit from risking 1+v in each market is:[7]

$$E(\pi) = f(x_A) + f(x_B) + 2\sum_{XA+}^{XB-} f(x) + (-v)\,(1 - f(x_A) - f(x_B) - \sum_{XA+}^{XB-} f(x)) \qquad (3)$$

The first two terms are the probability (and expected payoff) that the score equals one of the observed lines. Note that $f(x_i)$ will be zero if x_i is not an integer, since outcomes of games must have integer point differences. Also important to note is that a tie game is extremely rare in the NFL, so if a tie game is one of the possibilities under consideration in equation (3), that event will have a near zero probability. The third term represents the expected value associated with winning both bets. The last term is the loss associated with winning one bet and losing the other, in which case we lose one vig. Equation (3) can be simplified to:

$$E(\pi) = (1+v)f(x_A) + (1+v)f(x_B) + \sum_{XA+}^{XB-} f(x)(2+v) - v \qquad (4)$$

Using (4) and a "discretized" [8] normal distribution $N(0,12.91^2)$, we calculated expected profits for the following proposition: suppose that an unbiased line is announced, and due to the bookies in two markets acting merely as brokers, the line moves up in market A and down in market B.

Table I gives the expected return from risking \$1.10 (\$1 + .10 vig if you lose) to win \$1 in each city. Table I assumes that the starting line is an integer. Values will be slightly different if the initial announced line is not an integer. For example, if the true, expected point spread suggests that team A should win by 6, suppose we see the line in city A move to 8, and down to 4 in city B. If the actual point difference is 5, 6, or 7 then we will win both bets, with an estimated probability of 9.2 percent. If the actual point difference is 4 or 8, we will win one bet and lose nothing on the other, with an estimated probability of 6.1 percent. Otherwise, we win one bet and lose the other, for a loss of \$.10. Looking at Table I in row 2, column 2 tells us that for risking a maximum loss of \$.10, the expected profit from our gamble in this situation is \$.161. Table I will of course be symmetric, since a symmetric (Normal) distribution was assumed. Similar to what we approximated in Section 4, any time one sees a line in one market that is two or more away from the line in another market, an expected profit will be made.

Payoff Table for Arbitrage on Different Lines								
Number of points moved (down) in city B	Number of points moved (up) in city A							
		0	**.5**	**1**	**1.5**	**2**	**2.5**	**3**
	0	-0.097	-0.066	-0.032	-0.001	0.032	0.063	0.096
	.5		-0.035	-0.012	0.029	0.063	0.094	0.127
	1			0.033	0.063	0.097	0.128	0.161
	1.5				0.094	0.128	0.158	0.191
	2					0.161	0.192	0.225

Table I : Expected payoffs (profit) from risking $1.10 in each of two cities in order to win $1. For example, is the line moves up one point from the initial line in city A, and down 1 point in city B, the expected profit would be 3.3 cents.

6. Why arbitrage is appealing

There are several factors that make this method of betting appealing. No complicated data analysis is required for an individual. By simply observing two different lines, one can roughly approximate the expected return from Table 1. Also, bettors and bookies are acting rationally within their markets. The inefficiency that is being exploited stems from the lack one of single, national market. By taking advantage of different preferences in different geographical locations, we can make a profit. The person betting the arbitrage simply has to observe the lines and estimate a probability that the actual point spread will lie in the interval between the two lines. A significant amount of money is never at risk for the bettor under this strategy: the worst outcome for any one game would be to lose 10 percent of a losing bet, since the worst outcome is wining one bet, and losing the other. One final appealing attribute is that as the bookmaker's information on lines gets better (the variance of the line around the true point spread gets smaller), this technique becomes more profitable.[9] Simply put, this will increase the probability of the actual spread falling within some interval close to the mean expected spread.

Of course, there are also some obvious drawbacks to this strategy. It may be costly to find two bookies in two different cities to place bets with. This is especially true given that we are dealing in a region where gambling is illegal. Also, there is definitely a cost associated with acquiring information on the lines from these bookies. However, given the fact that many people derive pleasure from the act of gambling itself (and not necessarily the money made), the additional cost may not be too large for some. In addition, in today's gambling market it may be possible to arbitrage between a local bookie and an internet gambling house on occasion.

One final interesting use of arbitrage is to act as a hedge for a previously placed bet. If a gambler placed a bet on Team A, and later the line adjusts due to a star player on that team becoming injured, then the first bet would have a very low probability of winning. Ceteris paribus, placing a bet for or against Team A with the new line would have an expected negative payoff. However, given the fact that a bet was placed on Team A previously, also placing a bet against Team A would have an expected positive return as long as the line adjusted by two or more points.

As with all papers in economics, there are some additional practical considerations one must pay attention to that have been ignored in order to make the analysis tractable. One must consider possible heterogeneity in the variance of the distribution of score differences between certain pairs of teams. For instance, the variance for two strongly defensive teams may be less than that of two strongly offensive teams.[10] Another consideration is that some score differences are more common than others. Differences of three and seven points will tend to be more common than differences of say, two, six or eight.

Nevertheless, the theoretical potential for arbitrage opportunities does exist. The burning question remains: How often do arbitrage opportunities arise between two different local markets, or between a local market and a more centralized market like Las Vegas? Unfortunately, this specific question remains unanswered. Collecting accurate data on a largely illegal market is difficult, at best. The closest answer can be found by looking at lines published by various on-line bookies. While it is still illegal for most Americans to place bets at these establishments, their lines are easily verifiable. A recently developed website, findyourbet.com , has been constructed to allow users to simultaneously compare lines offered at different internet sports bookies. Personius (2002) has found that differences of ½ point between these online bookies are common, and differences of one to two points have occasionally been seen.

7. Vegas as an unbiased estimate

The biggest drawback to this strategy is the necessity of finding a partner in crime in another city, forming what might be called a *syndicate*. Suppose that two members of a syndicate work together, sharing information and coordinating bets when sufficient deviations from the Las Vegas line occur. Sometimes both bets win, and sometimes one bet loses. The syndicate could meet once each year to divide the profits. But, what if the syndicate did not divide up the profits? Obviously, it does not matter. If we know that over many bets, the syndicate would make a profit together, then independently, each member should expect to make half of the profit (in the long run). Simply put, one will make an expected profit if one ever sees a point spread offered which is at least one point different from the Las Vegas line. The Las Vegas line, being unbiased, conveys information about the probability of different outcomes. Thus, one can make money from simply betting one side of the arbitrage suggested earlier, while also eliminating the transaction costs associated with forming a syndicate. The only significant difference is that that amount of potential "risk" has increased because the worst outcome is now to lose the bet (e.g. $1.10) rather than a worst case of losing one vigorish (e.g. $.10 net for both bets).

This kind of proposition is almost precisely what Vergin and Scriabin(1978), suggested, and more recently Tryfos et al. (1984) and Badarinathi and Kochman(1996a) studied. Badarinathi and Kochman's strategy was to bet on the underdog when the point spread was greater than 5 points. This rule alone was no better than chance, providing winners 50.6 percent of the time. However, if one were to obtain an “advantage” over Las Vegas line of 1, 1.5, or 2 points, one would have seen a 53.52 percent, 54.78 percent, and 56.03 percent winning bets over the years 1984-1993, which are consistent with the theoretical findings in this paper. It is well known in the NFL gambling literature that in order to cover the vigorish, a gambler must win $\frac{11}{21} \approx 52.38$ percent of bets in order to break even. As mentioned previously, the key is not in the strategy chosen, but in the advantage gained from comparing lines.

Strumpf (2003) recently used data acquired from the Kings County (Brooklyn) District Attorney in raids on illegal bookmakers. This data contains lines issued by bookies in the New York area during the late 1990’s. He finds that when New York area teams play, that the line is shifted by approximately one point against the local team. This finding confirms the fact that it is possible to find profitable opportunities by betting against the team in a bookie’s home area. If the same holds true in other local markets across the U.S., then it is likely to be possible to form betting consortia as described in Section 5.

8. Arbitrage in odds

To be complete, the odds market for NFL games should be discussed. Until the 1940’s, betting on NFL games was done on an odds system instead of the current spread system. Currently, legalized betting on NFL games is still done using the odds system (called the “Money Line”), although it is much less common than using lines. Woodland and Woodland(1991) show that risk averse bettors will bet more under a line system than under an odds system.[11] They define an odds bet as the following:

Bet (1+c)n dollars to win (B-c)n dollars (5)

where c is some commission charged for betting.[12] For example(ignoring commissions), if Miami is favored over Denver 2 to 1, then a $1 bet on Denver will win $2 if Denver wins the game, and a $2 bet on Miami will win $1 if Miami wins the game. There are two things to take note of. First of all, ties are rare in NFL games. If a tie does occur, then the bets are neither won nor lost, so they are not important in the case of odds. Also note that if B = 1, this is roughly equivalent to a bet with a spread of zero.

Suppose that the odds reflect the true probability of winning, or

$$\text{P(winning the odds bet)} = \text{P(team wins)} = \frac{1}{(1+B)} \qquad (6)$$

If B is greater than one, then the team in question is an underdog. In the case above, Denver would be expected to have a $\frac{1}{3}$ chance of winning.

Pope and Peel(1989) look at Britain’s betting market for soccer and make some interesting observations. First, there are four main, independent betting houses.[13] Second, the odds are fixed several days before each game, and are not changed. They found one instance in a database of odds where profitable arbitrage was guaranteed. Below we will derive the necessary conditions on odds that would give positive returns from arbitrage.

Consider the following arbitrage strategy: Suppose you wish to bet so that if you win a bet the guaranteed payoff is $1. Bets in odds are such that if you bet n units, you bet (1+c)n to win (B+c)n. If in cities 1 and 2 there exist two different odds on one game, B1 > B2 ≥ 1, then we can choose bets:

(1+c)n1 to win (B1 - c)n1 (7)

(B2 + c)n2 to win (1- c)n2 (8)

We place one bet on the underdog, and place that bet in the city with the higher payoff(B1). We also place one bet on the favorite, and place that bet in the city where it is cheaper to place the bet(B2).

We must choose n1 and n2 so that each bet will win us $1 **plus** the amount of the other bet that we will lose. This fact gives rise to the following two equations:

$$(B1-c)n1=1+(B2+c)n2 \quad (9)$$

$$(1-c)n2=1+(1+c)n1 \quad (10)$$

The first equation states that our winnings from our bet on the underdog must equal $1 plus the amount that we bet on the favorite. The second equation states that our winnings from our bet on the favorite must equal $1 plus the amount that we bet on the underdog. Solving these two equations simultaneously for n1 and n2 gives:

$$n1=\frac{1+B1}{B1-B2-2c-CB2-CB1}, n2=\frac{1+B2}{B1-B2-2c-cB2-cB1} \quad (11)$$

The opportunity for profitable arbitrage will exist when n1 and n2 are positive. This simply puts conditions on B1 and B2 such that the denominator is positive. We have constructed the bets assuming that B1 is larger than B2. How much larger it must be is given by the denominator:

$$B1-B2-2c-cB2-cB1>0 \quad (12)$$

or:

$$B1>\frac{B2+2c+cB2}{1-c} \quad (13)$$

The difference in odds necessary is increasing in c. If c is zero, then B1 need only be slightly larger than B2. Woodland and Woodland (1991) found that on average, c is about .05.[14] Making this assumption, then:

$$B1>1.1053B2+1.053 \quad (14)$$

A graph of this relationship is depicted in Figure 3.

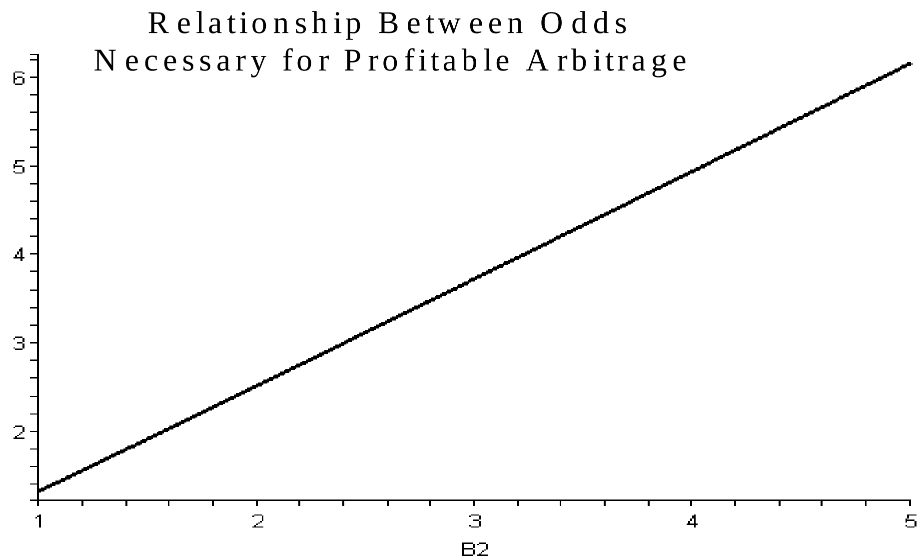

Figure 3: The relationship between odds necessary for profitable arbitrage assuming a 5 percent average commission rate (B1 as a function of B2)..

For example, if we observe the odds of 4 to 1 and 3.5 to 1 for the same game, a bettor could make as much money as he liked, with certainty. Of course, such bets are restricted by a bettor's available funds as well as the willingness of a bookie to take the bet. In this case, the .5 difference in odds is barely over the critical difference of .487. Using equation (11), we would need to buy 180 bets on the underdog, and 200 on the favorite. Thus, the total amount needed to win $1 with certainty is $899. However, if we see odds of 4 to 1 and 3 to 1, this amount decreases to $35.36.

Strumpf (2003) found evidence that sometimes local bookmakers set odds on baseball so that they themselves could arbitrage with Vegas odds. He found that this occurs when a gambler who is known to always bet on a certain team is given very unfavorable odds. This does not imply that the average bettor can take advantage of this, but does suggest that bettors loyal to one team can improve their profitability if they watch the Vegas odds and bet against the home team on occasion, perhaps making an offsetting bet using an online bookmaking service.

9. Conclusion

As Gray and Gray (1997) point out, "The existence of consistent statistical biases in point spreads is not, in itself, evidence of inefficiency. In the strict sense, market inefficiency requires that trading strategies can exploit biases to earn consistent profits." In other words, retrospectively rejecting a null hypothesis does not imply that gambling markets are inefficient. While many different strategies have been tested on NFL market data, few have measured up in out of sample tests.

This paper has shown that in some clearly observable cases, the gambling market is subject to arbitrage of a sort, buying two bets in different markets when at worst one will be lost, and perhaps both will be won. The main reasons this may occur are because of market separation, local bookmakers trying to act as brokers between gamblers, and the tendency of local gamblers to bet on the local team. Unlike well-organized markets, gambling must often occur in a thin, spatially separated market due to its illegality. Just how often arbitrage opportunities occur is an empirical question which may never have a good answer, due to the lack of data on these small markets. One might expect that as the gambling market on the internet becomes a larger share of the market, arbitrage opportunities may appear less frequently. Even if this is the case, as the local bookie's market becomes thinner, the internet may open up even more opportunities as discussed in this paper.

While arbitrage across markets was shown to be low-risk, it is also understandably costly because of information and other transaction costs. However, bettors can much more simply bet one side of the arbitrage whenever one sees a difference of one or more points between any local line and the Las Vegas line. We also saw that the lines in recent years appear to be better predictors of the actual outcome of a game than in previous years. If this is a continuing trend, the methods discussed will become even more profitable. Of course, there are costs involved with pursuing these transactions that have an expected profit. Whether or not the type of arbitrage highlighted in this paper measures up to an inefficiency in the market is left up to the reader's judgment.

ENDNOTES

1. Many studies have been done estimating the amount of illegal gambling. For a review of these studies, see Rosecrance(1988), Ch. 6.
2. Two possibilities are reducing alpha, or multiplying p-values by the number of tests performed (Bonferroni approach). Both approaches will increase the probability of a type II error, however.
3. Rather than change the line, bookies will often try to pool their risk by “laying off” some of the bets with another bookie if they receive too many bets on one team.
4. Bookies also sometimes change the vig to 5 percent or 15 percent to encourage or discourage bets on a certain team.
5. Data was from Marc Lawrence's Playbook (2000), a retrospective view of football statistics published yearly.
6. An F test determined that the variance has decreased from Stern's estimate (p=.0486). A chi-square test on the more recent data was unable to show that the data are different from a normal distribution with mean zero and standard deviation 12.91 (p=.277).
7. The notation summing from “XA+ to XB-” means “all possible score differences in between x_A and x_B”.
8. Finding the area under the normal curve surrounding each possible outcome over a range of one. For example, the probability of the game score being equal to the stated line is the area under the normal curve from -.5 to +.5.
9. That is, if we make the simplifying assumption that lines will still move at the same magnitude.

10. If we assume that a low scoring, defensive team averages scoring μ points with standard deviation σ, it might make sense to think that a team scoring 2μ points on average might have a standard deviation of $\sqrt{2}\,\sigma$, ceteris paribus.
11. For a given, positive expected return.
12. We will continue to use Woodland and Woodland's notation describing an odds bet. The reader should note that while odds bets are actually made in a different form, odds bets can easily be converted into one using this notation.
13. Some consolidation in this market has occurred since Pope and Peel wrote their paper.
14. As Woodland and Woodland(1991) point out in footnotes 2 and 3, a rough average commission for football is (was) .05, however, the rate of commissions for odds bets tend to rise as the odds rise. Recently, it is very common to see higher commission rates.

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