A DUOPOLY MODEL OF FIXED COST CHOICE

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1. INTRODUCTION

Comparison of firms in Cournot and Stackelberg equilibrium is a subject that has received much attention. A universally imposed assumption in most discussions of the Cournot and Stackelberg outcomes is that participants in markets are confronted with given cost structures. In some setups, like the models of Robson (1990), Anderson and Engers (1992) and Shaffer (1995), firms are assumed to have identical costs. In others, such as the Stackelberg model developed by Pal and Sarkar (2001) firms have different costs. However, the sequence of costs of firms choosing output is fixed, as is the level of costs once the equilibrium is obtained.

This assumption is, of course unrealistic, since firms invest considerable time and effort in cost cutting to either increase profit or market share. The purpose of the present paper is to study the impact of cost changes on firms in Cournot and Stackelberg equilibrium. We do this using a model similar that that of Neuman, Weigand, Gross, and Muenter (2001). The model assumes that firms can reduce marginal costs by investing in assets, thereby increasing fixed costs.

In Section 2 we set up the initial revenue and cost conditions facing the firms. For ease of exposition, a duopoly with linear market demand and cost functions is employed. In this section we present standard results for the Cournot and Stackelberg duopoly models.

In Section 3, we introduce the assumptions about the cost-changing possibilities for the firms and determine optimal fixed costs. We evaluate the impact of setting fixed costs at their optimal levels on profit and market share for the firms in the two models. Section 4 provides empirical evidence on the relationship between fixed and variable costs chosen by firms. Section 5 contains some brief concluding remarks.

2. A SIMPLE MODEL OF COURNOT AND STACKELBERG EQUILIBRIUM

Consider a situation with linear market demand produced by two firms. The inverse demand function will be given by $P = A – BQ = A – B(q_1 + q_2)$, where P is price, Q is quantity, and parameters A and B are both positive. Output is produced under two market settings.

First, we consider a Cournot model. For this model total costs for the ith firm, where i = 1,2, will be of the form $TC_i = FC_i + C_i q_i$, where FC is fixed cost and C is marginal cost. Costs are identical for the Stackelberg model. However, for ease of exposition the subscript i is replaced by

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L for the Stackelberg leader and F for the follower firm.

In the Cournot model profits for firm i are given by

$$Profits_i = (A - C_i)q_i - Bq_i^2 - Bq_iq_j - FC_i. \qquad (1)$$

Maximizing profits for the two firms and simultaneously solving the reaction functions results in the outputs:

$$q_1 = \frac{A - C_1 + C_2 - C_1}{3B}, \quad q_2 = \frac{A - C_2 + C_1 - C_2}{3B}. \qquad (2)$$

The resulting market shares are

$$M_1 = \frac{A - C_1 + C_2 - C_1}{A - C_1 + A - C_2}, \qquad M_2 = \frac{A - C_2 + C_1 - C_2.}{A - C_1 + A - C_2}. \qquad (3)$$

Profits for the two firms are

$$Profits_1 = \frac{1}{9B}\left[A - C_1 + C_2 - C_1\right]^2 - FC_1, \; Profit_2 = \frac{1}{9B}\left[A - C_2 + C_1 - C_2\right]^2 - FC_2. \; (4)$$

Equations (3) and (4) show the familiar results for a duopoly in Cournot equilibrium. Market shares and profits are proportional to firm costs, with the larger market share and higher profits going to the lower marginal cost firm.

For the Stackelberg model begin with the profits for the leader and follower firm:

$$Profits_L = (A - C_L)q_L - Bq_L^2 - Bq_Lq_F - FC_L, \qquad (5)$$
$$Profits_F = (A - C_F)q_F - Bq_F^2 - Bq_Lq_F - FC_F.$$

The follower's reaction function is derived from its first-order condition:

$$q_F = \frac{1}{2}\left[\frac{A - C_F}{B} - q_L\right]. \qquad (6)$$

Substituting (6) into the leader firm's profits and maximizing with respect to q_L yields the outputs for the Stackelberg model

$$q_L = \frac{A - C_L + C_F - C_L}{2B}, \; q_F = \frac{A - C_F + 2(C_L - C_F)}{4B}. \qquad (7)$$

Market shares for the Stackelberg firms are therefore

$$MS_F = \frac{A - C_F + 2(C_L - C_F)}{2(A - C_L) + (A - C_F)}, \quad MS_L = \frac{2(A - C_L) + 2(C_F - C_L)}{2(A - C_L) + (A - C_F)}. \qquad (8)$$

Profits for the Stackelberg firms are

$$Profits_F = \frac{1}{16B}\left[A - C_F + 2(C_L - C_F)\right]^2 - FC_F, \qquad (9)$$

$$Profits_L = \frac{1}{8B}\left[A - C_L + C_F - C_L\right]^2 - FC_L.$$

Equations (8) and (9) are similar to (3) and (4). They show that, similar to the Cournot model, in Stackelberg equilibrium a firm's market share and profits will increase when its marginal costs are lower than its rival's. However, in the market share equations a larger weight is placed on the margin of price over marginal cost for the leader firm, measured by the differential ($A - C_L$).

3. CHOICE OF FIXED COST AND BEHAVIOR OF THE FIRM

Our model of cost setting is constructed using a model similar to the one provided by Neumann et al. (2001). These authors suggest that if higher fixed costs are a result of new and improved production techniques, such costs may result in lower marginal costs. We therefore assume a relationship between fixed and marginal costs of the form

$$C = K_0 - K_1 * FC, \qquad K_0, K_1 > 0. \qquad (10)$$

The parameter K_0 defines the level of marginal costs without the cost-reducing investment FC, while K_1 is the decrease in marginal costs per dollar investment in fixed costs. Since marginal costs must be positive, equation (10) is relevant over the range of fixed costs $0 < FC < K_0/K_1$. Also note that the restriction $A > K_0$ must hold. This is equivalent to assuming that the equilibrium price must be greater than the marginal cost consistent with zero fixed costs. The assumption assures that price covers marginal costs under all possible situations.

All firms will be assumed to have access to the above technology, and will choose the profit-maximizing level of fixed costs using equation (10). This choice can be thought of as the second stage in a two-stage game, where the first stage is the choice of a profit-maximizing level of output. Although the Stackelberg model assumes a first-mover advantage for the leader firm in setting output, firms in both models will engage in simultaneous play at the second stage. This means that each firm will choose its profit-maximizing level of fixed costs given its rival's choice. The outcome of the second stage will be a *Nash equilibrium* in the choice of fixed costs.

Substituting (10) into (4) and (9) and differentiating with respect to FC results in the firm's profit-maximizing level of fixed costs. Substituting these costs into (3), (4), (8), and (9) yields the market shares and profits assuming the optimal levels of fixed costs are obtained. All results are displayed in Table 1[1].

Table 1: The Impact of Optimal Fixed Cost Choice by Type of Firm			
Firm Type	Optimal Fixed Cost	Resulting Market Share	Resulting Profit
Cournot	$\frac{K_0 - A}{K_1} + \frac{9B}{4K_1^2}$	$\frac{1}{2}$	$\frac{A - K_0}{K_1} - \frac{27B}{16K_1^2}$
Stackelberg Leader	$\frac{K_0 - A}{K_1} + \frac{13B}{6K_1^2}$	$\frac{3}{5}$	$\frac{A - K_0}{K_1} - \frac{5B}{3K_1^2}$
Stackelberg Follower	$\frac{K_0 - A}{K_1} + \frac{7B}{3K_1^2}$	$\frac{2}{5}$	$\frac{A - K_0}{K_1} - \frac{17B}{9K_1^2}$

The optimal fixed costs for the three firms contain the common factors $(K_0 - A)/K_1$ and B/K_1^2. Since 7/3 > 9/4 > 13/6, optimal fixed costs are highest for the Stackelberg follower and lowest for the Stackelberg leader, with fixed costs for the Cournot firm falling between these two extremes. This result is intuitively appealing. It suggests that since investment in capital expenditures reduces marginal costs with the potential of increasing profit margins, the incentive to do so is greatest for the firm in the most disadvantageous market position. A similar link between firm capitalization and firm positioning has been noted by Porter (1980, Chapter 15).

This choice in fixed cost results in equal market share for the two firms in Cournot equilibrium. This result is the same as the market shares obtained from equation (3) assuming $C_1 = C_2$, which is the standard result for a Cournot duopoly with equal marginal costs for the two firms and no choice of fixed costs. The result is a direct consequence of the firms having access to the same marginal cost reducing technology, and is the expected outcome of the simultaneous play at the second stage of the game. It suggests that no matter what the initial marginal costs, simultaneous play and access to the same marginal cost reducing technology at the second stage of the game will always allow the firm in a disadvantageous market position to "catch up" in market share[3]. This finding supports the apparent willingness for some firms to compete for market share as well as profits, since higher market share may be a more easily attainable goal than higher profits.

In the Stackelberg model the resulting market shares are 3/5 going to the leader and 2/5 of the market output going to the follower. This can be compared to the market shares for the Stackelberg model with equal initial marginal costs and no marginal cost reducing choice of fixed costs[4]. In this alternative situation the leader's market share will be 2/3 of the market, with the follower's share being 1/3. Since 2/5 > 1/3, in terms of market share the Stackelberg follower firm prefers the situation in which it can reduce marginal cost by investing in fixed costs to the situation in which its marginal costs are the same as the leader firm's. However, any potential market share disadvantage cannot be completely made up by choice of fixed cost as in the Cournot model. In essence, some of the first-mover advantage of the Stackelberg leader in choosing output remains with the leader in the second stage of the game.

Profits assuming fixed costs are at the optimal level are similar to fixed costs, both having the common factors $(A - K_0)/K_1$ and B/K_1^2. However, since 5/3 < 27/16 < 17/9, the ordering of profits is opposite that of fixed costs. Profits for the Stackelberg leader are largest. Profits for the Stackelberg follower are smallest. The Cournot profits lie between these two values.

The results for profits in the Stackelberg equilibrium are like those comparing market share. They show that the reduced profits resulting from moving second in a Stackelberg game cannot be eliminated by investing in fixed assets once equilibrium is obtained. The result is in the spirit of observations made by Porter (1980, Chapter 15), who argues that in attempting to catch up with market leaders follower firms have a tendency to over capitalize, thereby decreasing profits.

4. EMPIRICAL EVIDENCE OF COST CHOICE BY FIRMS

The key assumption of the above analysis is equation (10) which postulates a negative relationship between fixed costs and marginal cost. We tested this relationship as follows.

Data on 273 mid-capitalization firms were collected for the period 2002-2000 from the Compustat Data tapes. Accounting costs for these firms were categorized into fixed and variable costs. The variable cost (C) proxy used was Cost of Goods Sold. These costs included labor, heat, power, freight, and other costs directly involved with producing and distributing a product. Some administrative costs such as plant insurance were also reported under this heading. The fixed cost (FC) measure included non tangible fixed costs such as advertising and marketing costs, as well as fixed costs associated with tangible property. Examples of such costs would be tools and dies, software, and aircraft. Fixed costs were measured relative to total assets to avoid the impact of firm size on cost measurement.

Two functional forms were estimated.

$$\Delta C_t = \alpha + \beta_1 \Delta(\frac{FC}{A})_{t-1} + \beta_2 \Delta(\frac{FC}{A})_{t-2} + \beta_3 \Delta(\frac{FC}{A})_{t-3} \qquad (11a)$$

$$C_t \setminus C_{t-1} = \alpha + \beta_1 \Delta(\frac{FC}{A})_{t-1} + \beta_2 \Delta(\frac{FC}{A})_{t-2} + \beta_3 \Delta(\frac{FC}{A})_{t-3}. \qquad (11b)$$

Equation (11a) assumes a linear relationship between the current change in variable costs and lagged changes in fixed costs. The lags are included to allow for adjustment time. Equation (11b) expresses the change in variable cost in the form of the ratio of current variable cost to the previous period's variable cost. If equation (10) is a reasonable assumption, we should find significant negative values for the β_i in (11).

Table 2 displays these estimated coefficients. The t-statistics are in parentheses. The results are consistent with equation (10). The table shows a significant negative relationship between fixed and variable costs at the one period lag for the years 2002 and 2000, and a mild negative relationship at the three period lag for 2001. For 2002 the one period lag coefficient was significant when the dependent variable was expressed in both change and ratio form, while only the ratio form was significant for 2000.

When estimated over the entire 2002-2000 time period, a significant negative relationship between changes in fixed and variable cost was displayed at the one period lag. This relationship held when changes in variable costs were measured in both absolute and relative terms.

Table 2: Changes in Variable Costs Regressed Against Fixed Cost

Independent Variable	2002		2001		2000		2002 - 2000	
	ΔC_t	Ct/Ct-1	ΔC_t	Ct/Ct-1	ΔC_t	Ct/Ct-1	ΔC_t	Ct/Ct-1
$\Delta(FC/A)_{t-1}$	-1728.6***	-0.59****	128.9	-0.28	-70.3	-0.78***	-664.8**	-0.61****
	(-2.364)	(-3.138)	(+0.308)	(-0.979)	(-0.147)	(-2.408)	(-2.009)	(-3.839)
$\Delta(FC/A)_{t-2}$	273.1	0.47**	119.8	0.01	389.8	-0.07	288.9	0.20
	(+0.316)	(+2.114)	(+0.295)	(+0.021)	(-0.882)	(-0.224)	(+0.874)	(+1.235)
$\Delta(FC/A)_{t-3}$	-93.6	0.004	-163.6	-0.444*	-148.0	-0.42	NA	-0.26*
	(-.115)	(+0.023)	(-0.446)	(-1.8)	(-0.391)	(-1.624)		(-1.829)
R-squared	0.0219	0.0581	0.0018	0.0153	0.0047	0.0276	0.0069	0.0266

*Significant at the 10% level
**Significant at 5% level
***Significant at 2% level
****Significant at 1% level

In sum, increases in fixed costs enable firms to decrease variable costs. And, this relationship seems to occur rather rapidly, that is, within one to two years.

5. CONCLUSIONS

Most comparisons of firms in Cournot and Stackelberg equilibrium assume given cost structures. The current paper analyzes the impact of marginal cost reducing fixed investment on market share and profits of firms in Cournot and Stackelberg equilibrium. The setup can be conceptualized as a two-stage game. At the first stage firms choose their profit-maximizing output. Cournot and Stackelberg duopoly models were used at this stage. In the second stage, firms were given the option of reducing marginal costs by investing in fixed costs. For both models, firms engaged in simultaneous play at this stage.

We found the greatest investment in fixed cost was for the Stackelberg follower and the smallest for the Stackelberg leader. The Cournot firm's investment fell between the two extremes. Resulting profits mirrored these investments, with the smallest accruing to the Stackelberg follower and largest to the Stackelberg leader.

Perhaps the most interesting result was the difference in market shares resulting from the two models. In the Cournot model, if a firm started from a smaller market share due to higher marginal costs, its investment in fixed costs at the second stage allowed it to equalize its market share relative to the lower-cost firm. This was not true in the Stackelberg model. No amount of fixed investment allowed the follower to equalize its market share with the leader's market share. The same held true for profits. This suggested that the first-mover advantage of the Stackelberg model was carried by the leader into the second stage of the game.

These conclusions assume that firms can reduce marginal costs by investing in fixed assets. Empirical evidence supporting this assumption was provided.

ENDNOTES

1. Proofs of these results are available from the author upon request.
2. The set of feasible solutions requires that fixed costs are greater than zero. These parameter restrictions are also available from the author on request.
3. We are not certain the extent to which this conclusion is dependent on our use of a linear function linking fixed and marginal costs.
4. Assume $C_L = C_F$ in equation (8).

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