MICROECONOMIC MODELING AND ANALYSIS OF COMMODITY CHEMICAL PRODUCTION IN A SIMPLE PLANT

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ABSTRACT

A four-input (capital, labor, material and energy) production theory is applied to a representative chemical reaction occurring in a model plant, which captures the general features of large-scale chemical production processes. Engineering model and *bridge equations* (links between engineering and economic variables) are numerically solved to obtain feasible input combinations for a given production rate. Labor and material flows are fixed for a constant production rate, such that the capital-energy isoquant/isocost map gives the technically efficient region and (cost minimizing) optimum output expansion path for planned plants (*ex ante* case). Model plant total capital investment versus plant capacity is in excellent agreement with capital investment costs for actual polymerization plants. Finally, short- and long-run total, average and marginal cost curves exhibit theoretically correct behavior, and an example of static equilibrium analysis of the firm in the chemical product market is presented using short-run cost and postulated product demand and marginal revenue curves.

INTRODUCTION

Simple empirical economic models of industrial production processes are desirable from both applied and theoretical points of view. Models formulated in terms of a few meaningful variables (i.e., capital stock, and flows of labor, raw material, energy, and product output) facilitate analysis and assessment of alternative process technologies and yield useful general conclusions regarding process *trade-offs* and economies of scale. Such models provide useful tools with which to optimize processes, plan future plant additions, and to perform static equilibrium analyses of the firm in various market settings. A novel approach has recently been advanced (Gow and Gow, 2003; Gow 2003a, b; Gow 2002) to derive economic models from engineering models for single unit chemical production processes. This approach has yielded useful technical and economic information for the processes studied. This paper extends the same methodology to an entire chemical plant consisting of a chemical reactor and separation cascade to produce a common commodity chemical product.

BACKGROUND

A production function expresses the relationship(s) between the inputs of resources (i.e., capital, labor, material, etc.) to an industrial process and the output of a desired good. The general production

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function is often of the form

$$q = f(x_1, x_2, \ldots, x_n) \quad (1)$$

where q is the output flow (units/time) of the desired product, and x_n is the stock (units) or flow (units/time) of the nth input to the process. The particular functional form of equation 1 (which may actually consist of a set of equations) may be either: (1) chosen from a list of empirical models by statistical tests (econometric or *top-down* approach); or (2) derived analytically or numerically from the detailed production technology (engineering or *bottom-up* approach). The most basic form of empirical production function is the Cobb-Douglas two-factor model (Cobb and Douglas, 1928)

$$q = A\, k^{\alpha}\, l^{\beta} \quad (2)$$

where k and l are the capital and labor inputs respectively to the production process, plant, firm, industry, sector or economy, α and β are the capital and labor output elasticities respectively, and A is the "time-dependent" index of factor productivity. A simple (unitary elasticity of substitution) form of the Cobb-Douglas model is obtained if $\beta=1-\alpha$ is specified.

Variations of the Cobb-Douglas model and several more-flexible functional forms (Zellner and Ryu, 1998; Diewert and Wales, 1995; Bairam, 1994; Pollak and Wales, 1987; Arrow et. al., 1961) have been successfully used to correlate industrial or sectoral level data for various product groups including consumer durable goods, food, clothing, paper, agricultural products and chemicals. However, generalizations regarding returns to scale and substitution elasticities reached from the results of *time-series* and *cross-sectional* studies (Giannakas et. al., 2000; Hsieh, 1995; Sato, 1975) are statistically supported by industrial level data without documented support at the process level. Furthermore, while application of generic economic models to industries and sectors is somewhat useful, difficulties are encountered regarding the division and units of input factors of production, the method of aggregation from firm-to-industry level of production, and assumptions about returns to scale and technological progress.

A useful method for obtaining meaningful process or plant production functions is the engineering (*bottom-up*) approach (Gow and Gow, 2003; Gow 2003a, b; Gow 2002; Barsan and Ignat, 2001; Sav, 1984; Chenery, 1949). Here, a deterministic model is derived from scientific laws, process constitutive relationships, and aggregation rules (*bridge equations* which provide the link between engineering and economic variables). Input unit cost data are incorporated to obtain a complete economic model (production and cost functions) from which the optimum combination of inputs may be determined. This paper applies the above methodology to a typical chemical production process occurring in a simple plant. The reaction and simple plant studied have features common to a wide range of intermediate (commodity) chemical production processes, and can yield useful general microeconomic information about large-scale chemical production.

CHEMICAL PROCESS FUNDAMENTALS AND PLANT MODEL

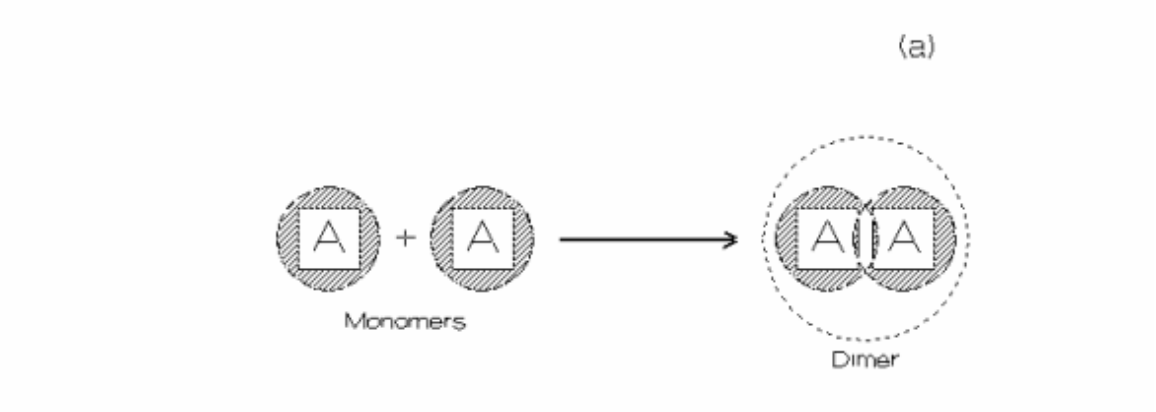
A broad spectrum of consumer products and intermediates (used in the production of other finished goods) are obtained from chemical production processes. These products, which account for nearly ten percent of the U.S. gross national product, include bulk commodity chemicals, pharmaceuticals, electronic materials, polymers and various other substances (Luyben and Wenzel, 1987). Chemical substances (elements and compounds) undergo reactions to produce new substances. *Chemists* are involved in the discovery and explanations (mechanisms) of chemical reactions, whereas *Chemical Engineers* are primarily engaged in the development of large-scale commercial processes to safely, ecologically and economically produce chemical products.

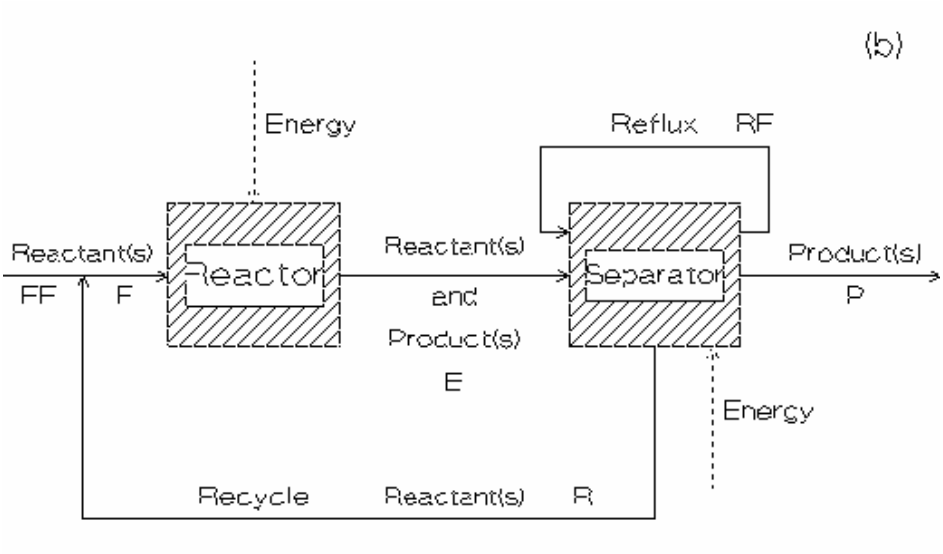
Industrial chemical production generally consists of two types of processing steps: (1) *reactions* - chemical reactants (reagents or raw materials) are converted to products; and (2) *separations* - products and unreacted raw materials are recovered in nearly pure form. Reactions are frequently incomplete (i.e., a fraction of raw materials remains and/or multiple products are produced); hence, separations are almost always required. Products are isolated, and unreacted reagents are recovered for further processing (reaction). Reactions take place in vessels called *reactors*, and separations occur in vessels called *separators* (or *separation cascades*). All plants consist of various types and combinations of reactors and separators. A very simple reaction and separation network comprised of one reaction unit and one separation unit is used here to illustrate the novel microeconomic modeling approach and the process and economic *trade-offs* typically encountered in industrial chemical production practice.

Model Reaction - Organic (carbon-based) intermediates are a large and vital segment of the chemical process industries (CPI). These substances are the building blocks for a wide range of end products including polymers, petrochemicals, and pharmaceuticals. Combination of small identical organic molecules to produce a larger intermediate molecule is a common class of industrial reactions. The simplest case of combination is *dimerization*, in which two molecules of reactant are combined to produce a product with twice the molecular mass of the reactant (see Figure 1a).

Model Plant - The model plant *process flow diagram* (PFD), which is shown in Figure 1b, captures the major technical features of many commercial chemical production processes. The model *dimerization* plant consists of a chemical reactor, in which raw material A is converted to product B, and a separation cascade (series of identical units called *stages*) which recycles the unreacted *monomer* (i.e., the unreacted raw material is recovered in the separator and sent back to the reactor for further processing). Reactant A enters the plant at a flow rate FF tons/day, mixes with recycle A at a flow rate R tons/day, and the combined stream enters the reactor at F tons/day (F=FF+R). However, only a fraction of the combined stream, f x F, is converted to the desired product B. Thus, the reactor effluent stream (stream E leaving the reactor) consists of a mixture of desired product B and unreacted reactant A. The separation cascade also has its own internal recycle stream, RF tons/day called *reflux*, the reason for which is discussed below.

Figure 1. (a) Dimerization reaction, and (b) simple plant showing components and material and energy flows.





There are a few key relationships, which describe the process behavior of the plant. First, total material is conserved. That is, there is no build-up or depletion of material within the plant over time. Thus, if 1,000 tons/day of monomer A enter the plant in stream FF, then 1,000 tons/day of dimer B are produced within the plant and must leave the plant in stream P. This is called the *law of conservation of mass*, which states that matter is neither created nor destroyed, but may be converted from one form to another (i.e., new substances may be formed in a chemical reaction). Another important relationship pertains to the chemical reactor, which facilitates intimate contact of reactant molecules required for reaction to occur. The longer that reactant molecules spend in the reactor, the higher the fraction of them that will be converted to product molecules. This is accomplished by either using a larger reactor or a lower feed flow rate, F, for a set production rate, P, of the dimer product. Finally, some general principles apply to the separation cascade. Here, higher flow rate through the cascade (throughput) results in more efficient (faster) separation and hence, requires a smaller separation cascade (i.e., smaller number of repetitive units or *stages*); however, a higher flow rate also requires larger diameter stages to

accommodate the increased flow rate. The internal flow rate (cascade throughput) is adjusted by increasing or decreasing the flow of stream RF. Note that increasing the flow rate RF has the effect of reducing the number of stages (more efficient separation) while increasing the stage diameter due to the increased throughput.

In summary, there is a key process trade-off for both the reactor and the separator. First, a smaller reactor may be used for a given dimer production rate, P (tons/day), which results in more A per day being recovered and recycled back to the reactor (higher R); or a larger reactor may be used, which results in lower R. The internal reflux of the separation cascade, RF, may be made large (low number of stages/large diameter) or small (high number of stages/low diameter) for a specified production rate, P. The essential long-run optimization problem is *what are the optimum sizes of the reactor and separation unit for a given production rate of B?*

ECONOMIC MODEL

The simplest production function for a chemical process is of the general form

$$q = f(k_T, l, m, h_T) \qquad (3)$$

where q is the product output flow (mass/time), k_T is the capital stock (i.e., reaction and separation units and plant infrastructure), l is the process labor flow (workerhours/time), m is the raw material input flow to the process (mass/time), and h_T is the total energy flow to the process (energy/time). Chemical process model equations may be combined with aggregation rules (*bridge equations*), which provide the link between engineering and economic variables. However, a fundamental problem encountered in the econometric (*top-down*) approach to production is how to aggregate heterogeneous capital equipment items.

Fortunately, engineering equipment cost correlations provide a consistent means for obtaining an accurate measure of the chemical plant capital stock using an engineering (*bottom-up*) approach. An equipment cost correlation expresses the purchased cost of an item as a function of the size of the item according to

$$C_{pur} = a\,(size)^b \qquad (4)$$

where C_{pur} is the purchased cost of the equipment item ($) at a specific point in time (base month/year), and the constants a and b are for a particular type of equipment constructed of materials to withstand specified extremes of operating conditions (temperature, pressure, corrosiveness of reagents, etc.). The installed cost of the equipment item in the base year, C_{ins} ($), is given by

$$C_{ins} = F_i\, C_{pur} \qquad (5)$$

where F_i accounts for the labor and materials involved in the installation of the item. Purchased equipment cost correlations are available from published sources[1] (e.g., Peters and Timmerhaus, 1991).

Capital stock should be expressed in physical units (capital units) rather than financial units (dollars) since capital stock is physical material. A method was proposed (Gow and Gow, 2003; Gow, 2003a, b; Gow 2002) for expressing heterogeneous capital equipment items in terms of a common "physical" unit, which allows easy aggregation of capital. This approach, used here, is now briefly reviewed. First, all conditions and equipment sizes are determined from the process model equations[2]. Next, the dollar value (in a base month/year) of an installed equipment item of specified size is related to the base month/year dollar value of an installed reference equipment item of specified size. A capital unit is defined as an arbitrary physical amount of a reference capital good, where it is assumed that the nominal prices of all capital goods change by the same proportion over time, such that the total capital is the same regardless of base month/year used.

The chosen (arbitrary) capital standard is that 1 capital unit is equivalent to 10 feet of 6-inch schedule 40 carbon steel welded pipe, which cost $122 in January 1990 (base month/year) (Peters and Timmerhaus, 1991). The January 1990 value of an installed capital equipment item, C_{ins} ($), is determined and converted to capital units using the conversion factor 1 capital unit equals $122 (Jan. 1990) or

$$k_i = 0.0082\ C_{ins} \qquad (6)$$

where k_i is the units of capital stock (capital units) for equipment item i. The total process capital, k_T, is then obtained from

$$k_T = 3.37\ \Sigma_i\ k_i \qquad (7)$$

where the multiplier 3.37 accounts for the necessary plant infrastructure (i.e., land, buildings and grounds, electrical and plumbing systems, controls, office and distribution facilities, etc.). The breakdown of plant infrastructure items for a typical chemical plant, which leads to the multiplier value of 3.37, is presented in Gow (2003b).

Plant labor requirement is also obtained from published correlations in the engineering literature. Published process labor studies (Guthrie, 1970; O'Connell, 1962; Haines, 1957; Isard and Schooler, 1955) propose that process labor for a chemical manufacturing operation depends on: (1) the total mass throughput, *m* (tons/day), (2) the *subdivision* or number of repetitive or distinct major process units, and (3) the degree of process *automation*. The model used here is built on the findings of these earlier studies and is given by

$$l = a'\ m^{0.25} \qquad (8)$$

where *l* is the process labor requirement (workerhours/day) and *m* is the process mass throughput (tons/day), and a' is a constant related to the degree of process subdivision (a' is higher for a larger number of process units) and the degree of process automation (a' is lower for a more highly automated process). Here, a'=11.0 for a moderately automated plant consisting of a few unique process units.

The total material flow to the plant (mass throughput) is simply given by the mass production rate of dimer B (i.e., since mass flow entering the plant equals mass flow leaving the plant)

$$m = q = FF = P \qquad (9)$$

Finally, the required energy input, h_T, for the plant includes electrical energy supplied to the reactor agitator and liquid pumps required to move fluids through the plant, and heat supplied to the separation cascade[3]. Furthermore, the fluid motion energy supplied to the agitator, pumps, and separator is "sunk" (lost) because this energy is not recovered in any useful form. Thus, the energy requirement for the model commodity chemical plant is

$$h_T = \Sigma_i h_i \qquad (10)$$

where *h* is in units of horsepower (HP). Application of Equation 10 requires constitutive equations, which express the terms h_i as functions of process variables[4].

RESULTS

Simulation Specifications and Methods – Long-run (planned plant) simulations were made for a large number of dimerization rates covering a wide range of production; however, detailed results are presented for three production rates (i.e., q=1,500, 2,000, and 2,500 tons/day), which cover the typically observed output range for real plants. Furthermore, one short-run simulation was made for the optimum plant (plant size of k_T=77,200 capital units) designed for a capacity of 2,000 tons/day of product. The *process model* is solved to determine the reactor size (volume) required to achieve a particular fraction, f, of the monomer A in the reactor feed stream F converted to dimer B product for a given pure A plant feed, FF (tons/day), and the separator size (i.e., number of stages and cascade diameter) required to achieve the desired B production rate, P (tons/day) (long-run or *ex ante* case); or to determine the production rate, P (tons/day), for specified reactor and separation cascade sizes and varying fresh A feed flow rate, FF (tons/day) (short-run or *ex post* case). Data produced in the simulation runs are presented in various useful forms and the following analyses are performed: (1) technical analysis - examination of long-run (*ex ante*) isoquant maps; (2) economic analysis - determination of the optimum output expansion path (minimum cost path across an isoquant map) and study of short- and long-run total, average and marginal cost curves; and (3) static equilibrium analysis - investigation of plant profitability in a typical short-run market setting.

Technical Analysis - Long-run technical analysis investigates the input factor relationships for planned plants and identifies the region of technically efficient production (i.e., decreases in at least one production factor and increases in other factors to maintain constant production rate). The optimum point in a technically efficient region is determined from an economic analysis of the process using unit factor costs. Technical analysis for the four-factor chemical process/plant model considers relationships (isoquant maps) between various pairs of the four input factors (i.e., capital stock, k_T, labor, l, material, m, and energy, h_T, flows). A fortuitous consequence of the simple chemical plant model is that labor and raw material inputs are constant for any constant production rate of B considered[5]. Hence, the capital versus energy isoquant map captures all of the essential features of the simple dimerization production process and yields valuable new information about the economics of chemical production.

The capital versus energy isoquant map for the model dimerization plant was constructed using data from a series of constant output sub-processes for which the fraction of monomer A reacted, f, was constant (e.g., f=0.30, 0.40, 0.50, etc.) and the reflux flow of monomer A in the separator, RF, was varied to obtain the number of separation stages, N_{act}, required to recover nearly pure A recycle and pure B product streams. Each set of data is for constant reactor size and energy input and variable separator size and energy input. Figure 2a illustrates the construction of a capital-energy isoquant from a series of constant conversion capital-energy *isoquantlets* for a dimer B production rate of q=1,500 tons/day[6]. *Isoquantlets*, shown for f=0.830 and 0.943, exhibit a small range of capital-energy substitution possibilities, and the envelope of *isoquantlets* defines the capital-energy isoquant for the given production rate, which in this case is q=1,500 tons/day.

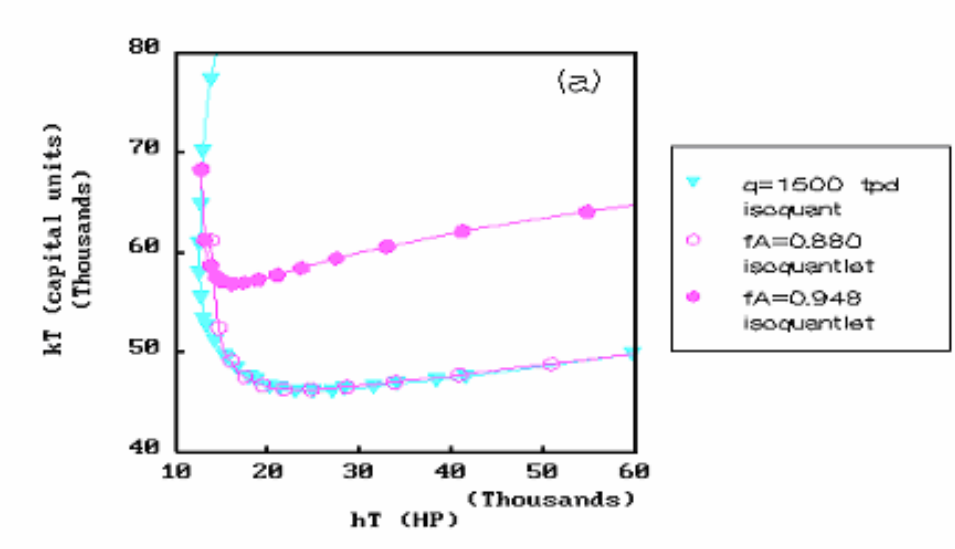
The *ex ante* capital versus energy isoquant map for the simple dimerization plant, including isoquants for q=1,500, 2,000, and 2,500 tons/day of dimer B output, is shown in Figures 2b and c. First, the isoquants have the theoretically correct convex-to-the-origin shape. Furthermore, the isoquant map exhibits ridge curves, which are the loci of points of zero or infinite slope [i.e., $(\partial k_T/\partial h_T)_q$=0 or ∞] on adjacent isoquants. The lower and upper ridge curves, which define the technically efficient region of production, have been added to the capital-energy isoquant map in Figure 2b. It is significant that that capital and energy are substitutes to a limited extent. Moreover, the isoquant map clearly shows that simple Cobb-Douglas or CES models are incapable of describing input factor relationships for the simple chemical plant studied here. These models assume a defined returns to scale behavior and monotonically declining marginal rate of technical substitution along an isoquant, which are not observed in the isoquant maps of Figures 2b and c. Similar results have been obtained in other studies of a variety of chemical production processes (Gow and Gow, 2003; Gow, 2003a, b; Gow, 2002), which especially support using the *engineering approach* advanced in this paper for any type of chemical process at the single unit or plant level. Finally, it is difficult to determine returns to scale behavior from the simulation results because it is not possible to simultaneously vary all inputs by the same proportion.

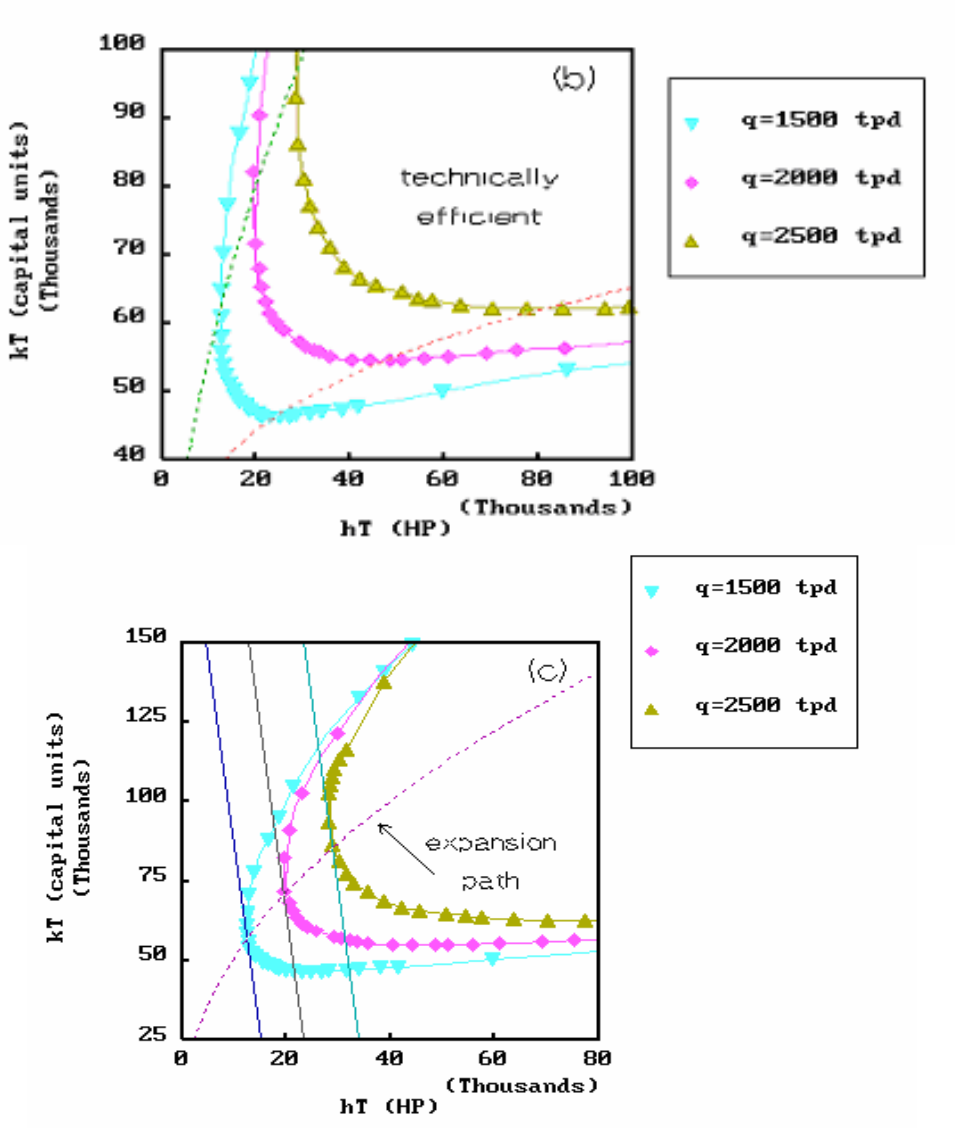
By far, the most useful feature of the capital-energy isoquant map is that it can be used to optimize the combination of inputs for planned plants at any level of production. A series of (parallel) capital-energy isocost (dashed) lines have been plotted on the capital-energy isoquant map in Figure 2c.

The unit costs of productive inputs (capital, labor, material and energy) are given the economic analysis section of this paper. The endpoints of the isocost lines are capital and energy equivalents, which may be purchased with a fixed budget after accounting for the constant required labor and raw material input costs at the given dimer B production rate. There are an infinite number of isocost lines of the same slope (each for a different total budget), which may be plotted. However, only the isocost lines that are tangent to the capital-energy isoquants for dimer B production rates q=1,500, 2,000, and 2,500 tons/day have been plotted in Figure 2c. The points of tangency between isocosts and isoquants give the economic optimum (cost minimizing) combination of inputs at each production rate. The optimum output expansion path (dashed curve in Figure 2c, which is the locus of least cost points) suggests that plant size (capital stock) should be increased at a lesser rate than energy consumption to increase production, based on current capital and energy unit costs. A change in either capital or energy unit cost would shift the location of the optimum output expansion path.

The observed capital-energy relationship in Figures 2b and c may be compared with findings from other published studies. Curiously, the relationship between capital and energy inputs in production is a widely studied unresolved issue in the economics literature. Berndt and Woods' (1979) seminal study suggests that capital and energy are complements (i.e., *kinked* isoquants with two positively sloped branches and zero substitution possibilities). The results of some studies support Berndt and Woods' claim. Mahmud and Chishti (1990) concluded that capital and energy are complements in Pakistani manufacturing, and Caloghirou et. al. (1997) found that electrical energy and capital are long-run production complements in the Greek economy.

Figure 2. (a) Construction of isoquant from *isoquantlets*, and isoquant map showing (b) technically efficient region, and (c) output expansion path.





However, other studies present evidence, which supports capital-energy substitutability. Rushdi's (1991) investigation of the South Australian electrical supply industry, Chang's (1994) study of Taiwanese manufacturing, and Hisnanick and Kyers' (1995) and Ziari and Azzams' (1999) studies of U.S. manufacturing conclude that capital and energy are substitutes. Finally, one paper proposes a capital-energy duality depending on the time frame (short- or long-run) considered. Apostolakis (1990) found that cross-section (long-run) studies support substitutability between capital and energy, whereas time series (short-run) studies support the capital-energy complementary hypothesis. An important result of this study is that capital and energy are clearly shown to be weak-to-moderate substitutes in the long-run. It is significant to note that all of the interesting input behavior in chemical production processes is for capital and energy. Capital and energy may both be varied (along isoquants) in the long-run, whereas energy is the only important variable input in the short-run.

Finally, it is useful to compare key benchmarks such as capital investment cost of model dimerization and real polymerization plants[7]. Figure 3 shows profiles of the total capital investment versus

plant capacity for model dimerization and commercial polymerization plants from which excellent agreement is observed. Thus, it appears that the engineering approach, presented in this paper, can be successfully used to estimate the capital investment requirements of new chemical plants.

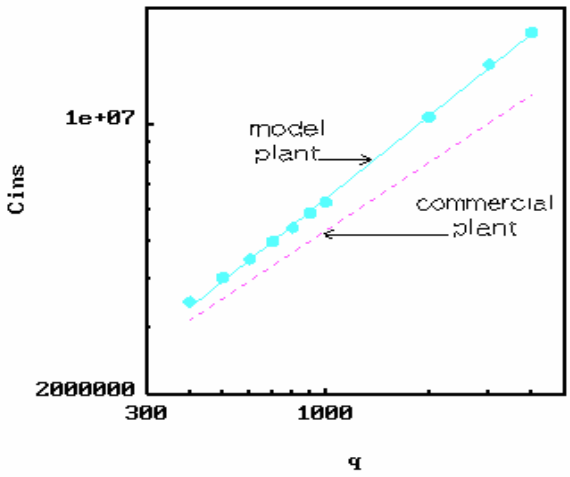
Economic Analysis – Economic analysis *optimizes* a process within the technically efficient region of production. Isocost lines are superimposed on the isoquant map to permit graphical determination of the optimum output expansion path in the two-dimensional case (two variable process inputs) as was previously illustrated for the simple dimerization plant. However, additional insight is gained by examining the behavior of both long- and short-run cost curves.

The total cost, TC ($/day), is given by

$$TC = C_{kT}\, k_T + C_l\, l + C_m\, m + C_{hT}\, h_T \qquad (11)$$

where C_{kT} ($/capital unit day), C_l ($/workerhour), C_m ($/ton) and C_{hT} ($/HP day) are the unit cost of capital,

Figure 3. Capital investment cost versus capacity for model and commercial plants.



labor, material and energy respectively. The values of these unit cost components are determined from practical considerations. The unit cost of capital, C_{kT}, is obtained by dividing the base month/year dollar value of the defined capital unit (1 capital unit equals $122, Jan.1990) by the average useful life of chemical process equipment, which is about ten years (3,600 days) (Peters and Timmerhaus, 1991). Moreover, the capital unit cost is corrected from 1990 to 2000 dollars using a common cost index, which accounts for inflation (Peters and Timmerhaus, 1991). Finally, the capital cost is multiplied by a factor of 1.10, which reflects the opportunity cost of investing in the next most attractive venture (assumed to yield an annual 10% return for a period of ten years). Thus, C_{kT}=0.0426 $/capital unit day represents the daily depreciation rate of total plant capital. The term $C_{kT}k_T$ in Equation 11 is the average daily cost of

maintenance and improvements necessary to keep the value of plant capital equipment constant at k_T. $3{,}600C_{kT}k_T$ is the dollar amount required to replace the existing plant when it is completely depreciated. The unit cost of labor, C_l=22.50 \$/workerhour, is the inflation corrected average of wage rates for semi-skilled plant operating and supervisory labor (Peters and Timmerhaus, 1991). C_m, is unit cost of reactant (monomer) A, which is taken to be \$500/ton in January 2000[8]. Finally, the unit energy cost, C_{hT}=0.80 \$/HP day, is the inflation corrected average electrical cost per kilowatt-hour for purchased and self-produced electricity (Peters and Timmerhaus, 1991).

Equation 11 may be applied to either the long-run (*ex ante*/planned plant) or short-run (*ex post*/fixed plant) case of production in the simple chemical plant. The unit factor costs, C_{kT}, C_l, C_m and C_{hT}, are used in Equation 11 to compute total cost, TC (\$/day) along isoquants (fixed dimer B production rate, q) for technically feasible input factor combinations (long-run case). Conversely, the short-run case involves determining the total cost, TC (\$/day), for the given plant size and feasible combinations of raw material, energy and labor input flows to achieve a given dimer B production rate. Here, the first term on the right-hand side of Equation 11 is constant due to the fixed plant size (i.e., reactor volume and number of separator stages). The long- or short-run average cost, LAC or SAC (\$/ton) respectively, is computed from

$$\text{LAC(SAC)} = (\text{TC})_{\min}/q \qquad (12)$$

where $(\text{TC})_{\min}$ is the minimum (optimum) total cost determined by considering all feasible input combinations at a particular production rate for either the long- or short-run case.

Results of the application of Equations 11 and 12 to the general long-run case and one short-run case of production in the simple chemical plant are presented in Figure 4a. Here, short- and long-run average cost, SAC and LAC (\$/ton) respectively, are plotted versus dimer B output (tons/day). Note that the short-run average cost versus log q for the fixed plant size (k_T=77,200 capital units for q=2,000 tons/day) is a narrow U-shaped curve, which is within and tangent to the wider U-shaped LAC curve.

The long- or short-run marginal cost, LMC or SMC (\$/ton) respectively, is the output derivative of the total cost

$$\text{LMC(SMC)} = \partial(\text{TC})_{\min}/\partial q \qquad (13)$$

The results of Equations 12 and 13 for LAC and LMC versus production rate, q, are plotted in Figure 4b, from which it is clear that the LMC intersects the minimum in the LAC from below as expected. The LAC is flat at just over 505 \$/ton for an output range extending from around q=700 to 2,200 tons/day of dimer B output, which indicates a considerable region of constant returns to scale for the simple plant.

Finally, short-run cost curves including the SAC and SMC (\$/ton) as computed from Equations 12 and 13 respectively, and the short-run average variable cost

$$AVC = (STC - C_{kT}k_T)/q \qquad (14)$$

($/ton), which gives the plant shutdown conditions, are computed for a plant of 2,000 tons/day capacity (i.e., a plant of k_T=77,200 capital units in size) and are presented in Figure 4c. Short-run average cost declines sharply and levels off at around SAC=505 $/ton over the production range from approximately q=1,500 to 2,300 tons/day. SMC is less than SAC at low output, is nearly constant in the range 503 to 505 $/ton from approximately q=1,500 to 2,300 tons/day, and intersects SAC at q=2000 tons/day from below, which is theoretically consistent. Furthermore, AVC falls sharply to values just one to two $/ton less than SAC for production rates ranging from around q=1,500 to 2,300 tons/day, and then nearly coincides with SAC at production rates above q=2,300 tons/day. It is noteworthy that the long- and short-run total cost curves, LTC and STC ($/day) respectively (not shown) are economically consistent. Both profiles have the theoretically correct shape and are tangent at the same output as for the average cost curves in Figure 4a. Furthermore, the zero-output total cost of a plant with a capacity of 2,000 tons/day is 3,290 $/day, which is the fixed cost of keeping the plant open with no production.

Static Equilibrium Analysis - It is useful to conduct simple equilibrium analyses of a typical industry firm in a typical market setting using derived cost curves and estimated product demand and marginal revenue curves. The analysis presented here is for a firm which holds a portfolio of equally sized plants of capacity q=2,000 tons/day (k_T=77,200 capital units). The short-run cost curves from the previous section (Figure 4c) are re-plotted along with estimated short-run demand (average revenue or AR) and marginal revenue (MR) curves in Figure 5. The equation of the linear demand curve is

$$P = a + bq \qquad (15)$$

where the slope (b=-0.1366 $ day/ton^2) and the intercept (a=1,011 $/ton) were determined by considering a plant with a typical 50% markup over average cost of production at the equilibrium output. Furthermore, it follows that the marginal revenue curve, MR ($/ton), is given by the relation

$$MR = a + 2bq \qquad (16)$$

where the slope is twice the negative magnitude of the demand curve slope.

The intersection of the marginal cost (MC) and marginal revenue (MR) curves gives the optimum (profit maximizing) rate of output for the fixed plant, which is q=1,850 tons/day (92.5% capacity utilization of a q=2,000 tons/day capacity plant). The plant profit (approximately 462,500 $/day) is given by the difference between the demand curve, AR ($/ton) and short-run average cost curve, SAC ($/ton), multiplied by the output at which MC intersects MR (q=1,850 tons/day).

Figure 4. (a) Construction of long-run average cost curve, (b) long-run cost curves, and (c) short-run cost curves for a plant capacity of *q*=2000 tons/day.

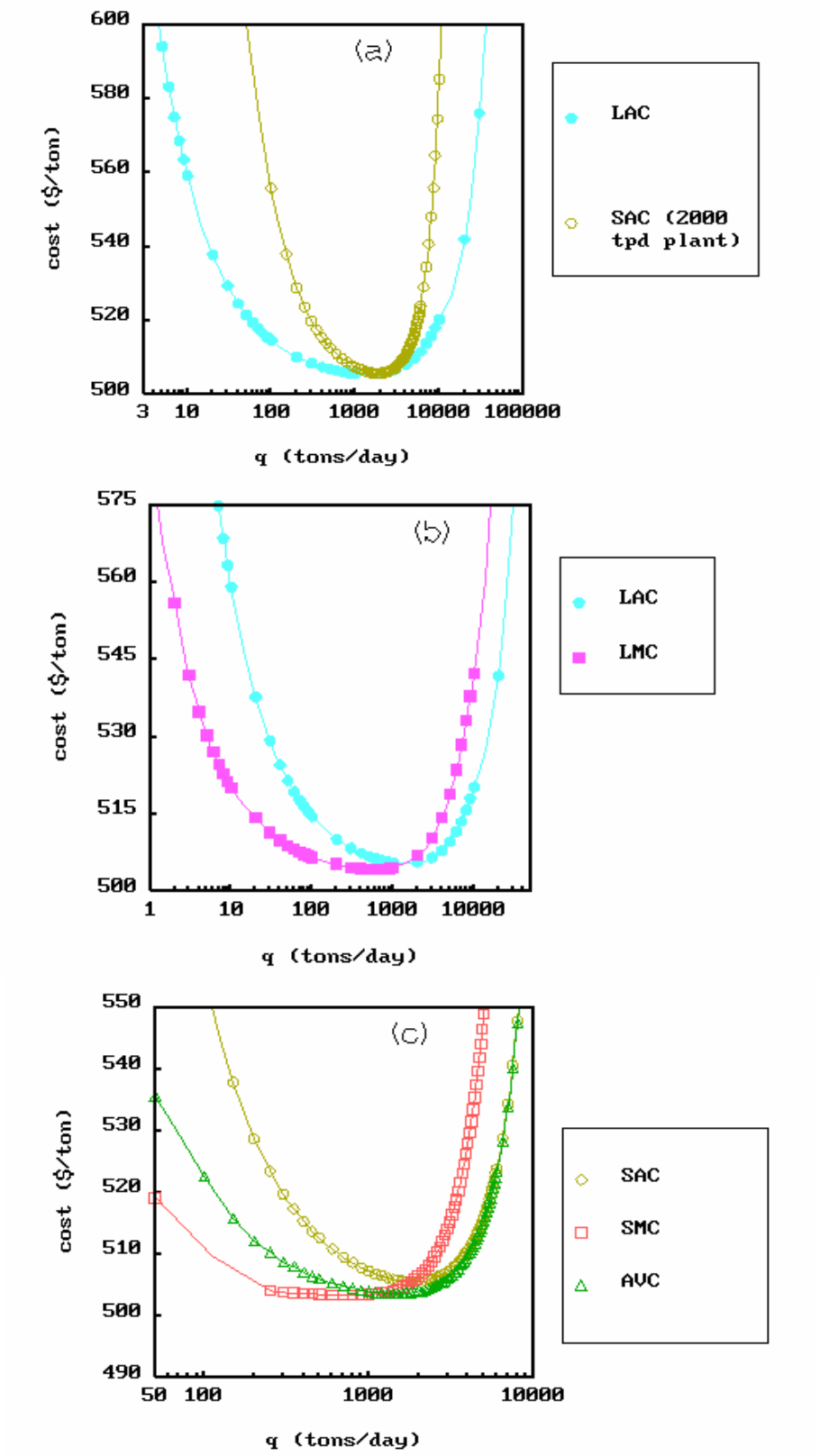
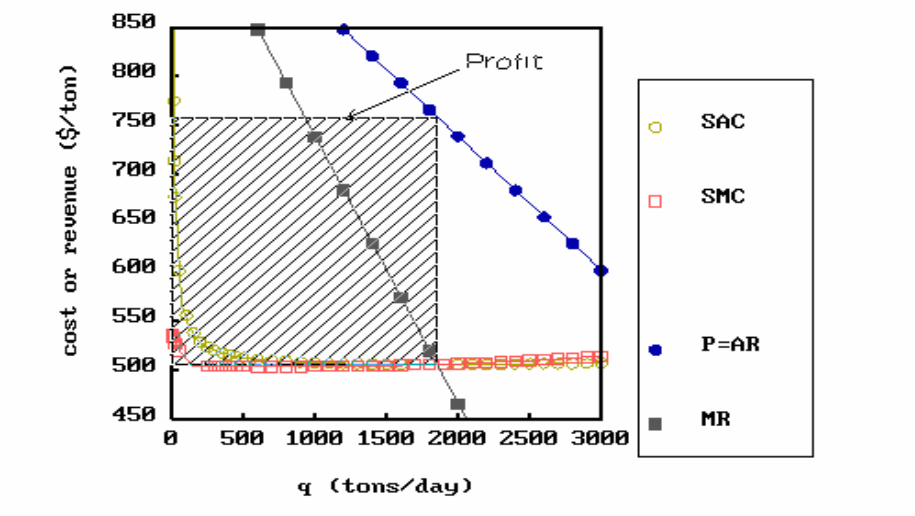


Figure 5. Static equilibrium analysis of a firm holding equally-sized plants of capacity *q*=2000 tons/day in an oligopolistic market setting.



CONCLUSIONS

A novel microeconomic framework was applied to a representative simple chemical plant for the production of a common commodity chemical substance (i.e., an intermediate, which is a raw material in the production of other chemical substances and finished goods). A four factor (capital, labor, material, and energy) production model was derived for a simple dimerization plant consisting of a chemical reactor and a multistage separation column. Labor and material inputs are constant for a fixed production rate, such that the capital-energy isoquant map gives all relevant input behavior for long-run (planned) plants. Simulation runs were made for both short- and long-run cases, and fundamental technical, economic and static equilibrium analyses of the results were presented. Useful results include identification of the technically efficient region for capital and energy inputs, the output expansion path and capital investment cost versus capacity for planned plants, the cost behavior of both fixed and planned plants, and the profitability of a firm comprised of equally sized plants in a typical market setting.

The observations made regarding technical, economic and static equilibrium analysis suggest some general conclusions for the economic behavior of chemical process and plant models. First, apparent *ex ante* capital-energy substitution possibilities reconcile the large body of conflicting published evidence on this topic, for which both input complementarity and substitutability are frequently reported. Moreover, the behavior of short- and long-run average and marginal cost curves is economically consistent and indicates economies of scale for the simple dimerization plant. In sum, the methodology applied in this paper provides chemical manufacturing firms with an essential tool for making decisions regarding the type, size, and timing of capacity additions. It is possible that the concisely developed theory at the plant level will ultimately facilitate simple aggregation to describe production relationships at the firm, industry, and sector levels.

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ENDNOTES

1. Cost correlations are periodically updated to account for both technological changes and inflationary effects. Inflationary effects are accounted for using a cost index, which relates the purchased cost of a bundle of capital goods in the present year to the purchased cost of the same bundle in a specified base year.
2. The detailed mathematical description of the *engineering process model* for the simple chemical plant is beyond the scope of this paper; however, a complete listing of model equations for all process equipment, the process variables and parameters and their units, the method of process model solution, and executable software are available from the author upon request.
3. The separator here is a fractionator in which heat is supplied to produce a vapor, which is enriched in the lighter monomer A component.

4. The author will provide a list of the constitutive equations (and assumptions) for each contribution in Equation 10 upon request.
5. See Equations 8 and 9.
6. An *isoquantlet* is the capital versus energy relationship for a special case of constant output, in which the reactor size is held constant while the separator size is varied.
7. Dimerization is the combination of two monomer molecules to form one dimer molecule, whereas polymerization is the combination of n monomer molecules to form one polymer molecule (n-mer).
8. The estimated monomer cost of 500 $/ton is the typical cost of raw materials used in polmerization processes.