A CORNER SOLUTION: COMMODITY FUTURES, DEFAULT FINES, AND UNINTENDED CONSEQUENCES

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JEL classification: *G14*

ABSTRACT

We analyze one specific form of market manipulation - corners (or squeezes) in some commodity futures markets. In the analysis we focus on the role of institutional factors such as the storage capacity at the delivery point and the severity of fines imposed on defaulting shorts. We analyze the influence of these factors on the likelihood that a corner-style manipulation might occur.

A reduction in storage capacity or an increase in the amount of fines imposed on defaulting shorts increases the probability of the occurrence. However, the latter factor seems quiet paradoxical. A high level of fines in case of default on a futures contract should decrease the number of defaults and thus make a futures contract more reliable. But, it tends to make the corners more frequent and thus makes the futures contracts less useful for the hedgers.

Introduction

We analyze one specific form of market manipulation - corners (or squeezes) in some commodity futures markets. A corner may be defined as making contracts for the purchase of a commodity, and then taking measures that make it impossible for the seller to fill his contract, for the purpose of extorting money from him.

For this purpose, a squeezer takes a considerable long position with futures contracts and simultaneously buys huge quantities of the physical commodity on the spot (cash) market. In this way the squeezer is able to gain control (at least partially) of the available supply of the given commodity. At the maturity of the futures contract, the manipulator call for delivery of the commodity. Some of the shorts are not able to deliver the required commodity because they do not own it and the only person from whom they can get it is the dominant long - the manipulator. The "squeezed" shorts are therefore obliged either to accept any price set by the manipulator or to default on their contracts and pay a heavy fine set by the futures exchange governing body.

This type of manipulation was fairly frequent on the floors of commodity futures exchanges in the second half of the nineteenth century - especially on the Chicago Board of Trade (the CBOT).

In our historical analysis, we focus on the role of institutional factors such as storage capacity at the delivery point and the severity of fines imposed on defaulting shorts. We analyze the influence of these factors on the likelihood of occurrence of a corner-style manipulation.

In the first part of our paper we present a few cases of real corners dating from 1860 to 1885.

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Some of them, such as the corners orchestrated by B. Hutchinson in 1866 and in 1888, were very successful. Others, such as J. Lyon's in 1872 led their initiators to bankruptcy and complete ruin. In the second part of the paper, we try to model the mechanism of corners as a game between the manipulator and the hedgers - sellers of the contracts. This type of modeling tries to make existing models, those of Kyle(1984) and Pirrong (1995a) for instance, more realistic. These models emphasize the interplay between a manipulator and the market makers, which, in our opinion does not exactly reflects the reality.

Our first result is the absence of the pure strategy Nash-equilibria, which leads us to consider mixed strategy equilibria. This approach helps us to interpret the results in terms of probability. We can therefore compute the probability that a manipulator will attempt a corner and the probability that hedgers will enter in the futures market.

Our results are fairly intuitive. A reduction of storage capacity or an increase in the amount of fines imposed on the defaulting shorts increases the probability of the occurrence of corners. The latter factor seems paradoxical. The high level of fines in case of default on a futures contract should normally decrease the number of defaults and then make a futures contract more reliable. But, it tends to make the corners more frequent and thus make futures less useful for hedgers.

1.FEW HISTORICAL EXAMPLES

The nineteenth century could be described as the "golden age" of commodity futures manipulators. The exchanges' internal rules were very permissive (e.g. no position limits) and the directors boards were very unlikely to take important measures to curb any nascent corner. Furthermore, the regulatory constraints were virtually non-existent. [The first binding act of U.S. Congress –the Commodity Exchange Act - was enacted in 1922]. Thus, during this period we can observe the interactions of purely economic factors with no regulatory intervention. In this section we describe three cases[1] of attempted manipulation reported by W.G.Ferris (1988) and J.W.Markham (1987). Our intent is to provide some factual evidence of corners (two successful and one failure) in which institutional factors played an important role.

1.1. BENJAMIN P. HUTCHINSON IN 1866

Benjamin Hutchinson led one of the first attempts to manipulate the Chicago wheat markets. His goal was to squeeze the August 1866 wheat contract. Benefiting from weak harvest forecasts he built up a considerable long position in the cash grain and in futures contracts in May and June 1866. The average purchase cost of wheat was reported to be around 88 cents per bushel.[2] In August the price rose steadily following the reports of weak harvests in Illinois, Iowa and other states tributary to Chicago. On August 4, the wheat contract was quoted at $ 0.90 -$ 0.92.

On August 18, Hutchison's demands for delivery raised the wheat price to $1.85-$1.87 causing the shorts huge losses. This corner and the other squeezes that followed Hutchinson's example prompted the directors of the CBOT to proclaim such activities illegal. They gave the first definition of a corner determining it as the practice *« of making contracts for the purchase of a commodity, and then taking* *measure to render it impossible for the seller to fill his contract, for the purpose of extorting money from him ».* They deemed such transactions improper and fraudulent, and declared that any member of the CBOT who engaged in this type of transaction should be expelled from the board. These declarations, however, had no effect on actions undertaken by some traders in the following years. Some of these attempted corners did not succeed and led their initiators to bankruptcy and ruin.

1.2. JOHN LYON IN 1872

On October 6, 1871 a spectacular fire, known as "the Great Fire", destroyed a large part of the city of Chicago. Six out of seventeen regular[3] grain elevators burned down which considerably reduced the storage capacity in Chicago from around 8 million bushels to 5.5 million. An important wheat merchant, John Lyon, felt that it was a good moment to launch a corner on wheat. He formed a coalition with Hugh Maher, another grain dealer and P.J. Diamond, a CBOT broker.

In spring 1872 the group started buying wheat (physical and futures). The price of wheat kept rising during the spring and the August contract sold at the beginning of July for between $1.16 and $1.18 a bushel. It reached $1.35 by the end of the month. This price rise caused a massive expansion of wheat arrivals in Chicago. They averaged 14,000 bu. per day at the beginning of July and then steadily rose to 27,000 bu. a day during the first week of August. During this week, on August 5, one more elevator, "the Iowa Elevator", was destroyed by a fire, further reducing Chicago's storage capacity (by 300,000 bushels).

Furthermore, bad weather reports reinforced rumors that the new crop would mature too late for delivery against August contracts. This added to the buying pressure causing the contracts to reach $1.50 by August 10 and $1.61 by August 15.This was the peak of the operation.

News of such high prices in Chicago caused farmers to greatly accelerate the harvest. W. C. Ferris (1988) reports lanterns carried via railroads to enable farmers to harvest the grain at night. On their way back to Chicago the trains transported ever greater amounts of wheat. In the second week of August the daily arrivals averaged 75,000 bushel and on August 19 they reached the unexpected and astonishing level of 172,000 bushels. For the rest of August, estimated daily arrivals were between 175,000 and 200,000 bushels. At the same time, the normal commercial channels were reversed. Usually, wheat from Chicago was shipped via Buffalo to the West Coast cities. Because of shipping costs the wheat price in Buffalo was generally higher than in Chicago, but in August 1872 the Chicago price was high enough to make shippers transport wheat from Buffalo back to Chicago to sell it to Lyon.

In fact Lyon had to keep the price high, and therefore had to keep buying the grain coming to Chicago in order to make his corner succeed. But the amount of wheat coming to Chicago greatly exceeded his anticipations and his financial resources. He was then obliged to raise more money with local banks. Chicago bankers were unwilling to lend him additional resources. Furthermore new elevators, constructed after the Great Fire started operating. It was estimated that the storage capacity was raised to 10 million bushels, a level 2 million bushels higher than before the 1871 fire. This further stretched the financial resources of the Lyon's group.

Lyon kept buying the grain till the afternoon of Monday, August 19, but he stopped when he learned of the banks' refusal to support him. The price of wheat immediately fell 25 cents a bushel. On August 20, Lyon announced the breakdown of the corner which caused an additional price decrease of 17 cents per bushel. This collapse ruined J. Lyon, who was unable to redeem his debts. P. J. Diamond destroyed his books and disappeared.

Other corners, however, did not end with the ruin of their initiators. The corner launched by Benjamin Hutchinson in 1888 is considered one of the most successful corners ever attempted.

1.3. BENJAMIN P. HUTCHINSON IN 1888

In the 1880s Benjamin Hutchinson was a very respected trader. He was distinguished with the nickname "Old Hutch". In spring 1888 he began to accumulate physical wheat and futures contracts calling for delivery in September. It was estimated that he was able to accumulate the contracts and the wheat at a price averaging 87-88 cents per bushel. Hutchinson also controlled the biggest part of the wheat in store in Chicago. (Storage capacity was then estimated at around 15 million bushels.)

Hutchinson's buying was met by a group of professional traders following a "percentage short selling" strategy. They were selling at the start of each crop year in the hope that the price decline at harvest time would be sufficient to cover their short positions with profit. John Cudahy, Edwin Pardridge and Nat Jones were among the biggest short sellers. The September wheat contract market rapidly turned to be a battle between Hutchinson and the short sellers.

Until August the price of wheat was steady at around 90 cents per bushel. But then reports came to Chicago that a heavy frost had destroyed a big part of harvest in the northwest. At the same time the estimates of Europe's needs turned out to be very pessimistic. Europe's deficit was thought to be at around 140 million bushels creating a huge demand for the wheat, owned in large part by Hutchinson. In these circumstances the price of wheat kept rising and on September 22 it reached the psychologically important level of one dollar. Nevertheless, the shorts kept selling in the hope that Hutchinson would not be able to absorb the growing amounts of wheat and then retreat causing the prices to decline. Yet, Hutchinson kept buying the grain and the contracts.

On September 27 (three days before the contract expiration) September wheat opened at $1.05, kept rising during the day and reached $1.28 causing a panic between the small shorts who begged Hutchinson to sell them some of the contracts he owned. At that moment his market power was complete.[4]

He decided to offer them 125,000 bushels at $1.25. W.G. Ferris (1988) reports : *« "Let them have what they want at $1.25", Old Hutch told one of his brokers, John Brine. The shorts stood in line before Brine, who handed out 125, 000 bushels at $1.25. »*

Hutchinson had at that time the power to set any price for the contracts. On September 28 he fixed the price at $1.50 but the biggest shorts refused to settle. Next day (the last trading day) he then set the price of the contract at $2.00 which became the settlement price.

W. G. Ferris (1988) estimates that around a million bushels were delivered to Hutchinson and that another million bushels were in default. Given the average buying price we estimate his total profit from this operation at around $1.5 million.

The frequency of corners and the relative similarity of the strategies of different types of traders makes modeling of this phenomenon valuable. We present these situations as a game between the manipulator and the sellers. It is convenient to use this setup to show the role of some institutional factors, such as storage capacity and the default penalties, in the build up of squeezes.

2. MANIPULATION AS A GAME BETWEEN HEDGERS AND MANIPULATOR

2.1 DESCRIPTION OF THE MODEL

Three types of actors are involved in the model: the manipulator, the hedgers and the market makers, but only the first two act strategically. The game is an interplay between two players the manipulator and a representative hedger. This type of modeling tries to make existing models (Kyle(1984), Pirrong (1995a) more realistic. They put the emphasis on the interplay between a manipulator and the market makers, which, in our opinion, does not exactly reflect the reality.

The manipulator decides whether to launch the corner or not while the potential seller of the contracts decides whether to hedge with futures or not. The market makers set the price of the contract as a function of the observed order flow. The trading spreads over two periods. In period 0 the players decide whether to trade futures or not and in period 1 they observe the consequences of their strategies. We assume that there is only one delivery point where the storage capacity is limited to S. Furthermore we assume that, in case of the corner, the maximum price the manipulator can ask is limited to D which is the level of penalty imposed on the defaulting shorts by the futures exchange authority or other body governing futures trading.

2.1.1 THE PRICE

We assume that the commodity cash price in period 1, denoted by P_1^C is normally distributed with the expected value $E(P_1^C)$ and the variance σ^2_p:

$$E(P_1^C) \rightarrow N(E(P_1^C)), (\sigma^2_p)$$

The market makers' role is to set the price of contracts in period 0, P_0^F. This price equals the expected price in period 1 and is a function of the observed orders flow. We assume that the market makers are able to detect the presence of a manipulator when hedgers are not present in the market. Then they detect that the corner is launched and set the price at the period 0 at D - the same level as the manipulator asks in period 1. If they do not detect the presence of the manipulator they set the price in period 0 as a "normal" expectation of the price prevailing in the spot market in period 1 namely $E(P_1^C)$[5]:

$$P_0^F = \begin{cases} D & \text{if manipulator is in the market} \\ E(P_1^C) & \text{otherwise} \end{cases}$$

2.1.2 THE MANIPULATOR

We assume that the manipulator is risk-neutral. She has the choice between two strategies: to manipulate (*M*) or not to manipulate (*NM*), in which case she does not enter the market. If she does not manipulate, her profit, denoted by Π_M, is nil. Otherwise, in order to manipulate she has to buy in period 0 a large number of futures contracts *X* exceeding the storage capacity *S* at the point of delivery. Then she asks for delivery of *X* contracts. The maximum amount the shorts are able to deliver is limited by the storage capacity *S*. The manipulator takes delivery of this amount of the commodity at the cost of P_0^F and resells it on the spot market after delivery at the price P_1^L . She offers to sell the remaining contracts at a price of *D*. Her expected profit is therefore equal to:

$$E(\Pi_M)= (X - S)\ (D - P_0^F) + S\ P_1^L - S\ P_0^F$$

where:

$(X - S)\ (D - P_0^F)$	- profit from the defaulting shorts
S P_1^L	- revenue from the sale of the commodity delivered to her
S P_0^F	- cost of the delivered commodity

Furthermore we assume that the revenue from the sale of the commodity delivered to the manipulator SP_1^L is lower, due to the "burying the corpse effect", than the cost of the delivered commodity S P_1^L < S P_0^F.

For the sake of simplification we assume that the storage costs and the discount rate between the two periods are nil.

2.1.3 THE HEDGER

The hedgers are the producers of the commodity. Each of them produces *Q* units. The production costs are nil.

We assume that they are risk averse with a constant risk aversion coefficient *2a*. Their expected utility function is:

$$U(\Pi_H) = E(\Pi_H) - aVar(\Pi_H)$$

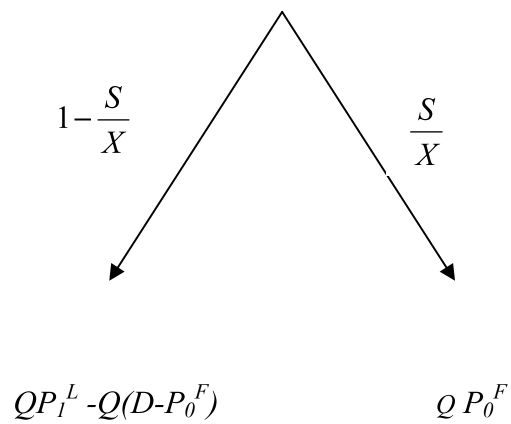
They have the choice between two strategies: to hedge (*H*) or not to hedge (*NH*) If they do not hedge their expected utility equals:

$$U(\Pi_H) = QE(P_1^L) - a\ Q2\sigma^2_p$$

If they hedge and if there is no corner their profit is constant and equals to QP_0^T which is also their expected utility. If they hedge and if there is a corner launched they can face two situations:

1. either they are able to deliver the commodity (with the probability *S/X* which depends on the proportion of the contracts held by the shorts which can be settled by the physical delivery out of the total deliveries required by the manipulator)
2. or they cannot deliver and then default because of the saturation of storage capacity. (with the probability *1-S/X*)

Their profit can be shown as follows:



The hedger's expected profit is:

$$E(\Pi_H) = \left(1-\frac{S}{X}\right)\left(QP_1^L - QD\right) + QP_0^F$$

and its variance is:

$$Var(\Pi_H) = \left(\frac{S}{X} - \left(\frac{S}{X}\right)^2\right)\left(QP_1^L - QD\right)^2$$

2.2. THE GAME

In the game, the strategies of two players, the manipulator and the representative hedger, are confronted. Both choose their strategies simultaneously. The profit of the manipulator depends on the price of the contracts in period 0 (P_0^F). If this price is low enough the manipulator's profit is positive:

$E(\Pi_M) = (X - S)\ (D-P_0^F) + S\ P_1^L - S\ P_0^F$

The contract's price in period 0 depends on the presence in the market of the two categories of players. If both are present, market makers are not able to detect the presence of the manipulator and thus the price set by them equals the expected spot price in period 1:

$P_0^F = E(P_1^C).$

If hedgers are absent from the market and if the manipulator is present, the market makers observe strong demand for the contracts and deduce that the corner is being launched and set the price for the contracts at the level of the default penalty - the price that will be demanded by the manipulator at the period 1:

$P_0^F = D.$

Then the manipulator makes a loss (due to "burying the corpse effect"):

$E(\Pi_M)= (X - S)(D- P_0^F) + S P_1^L – S P_0^F < 0.$

If the commodity producer does not hedge his expected utility equals:

$U(\Pi_H) = Q E(P_1^C) - a Q^2\sigma^2_p$

which could be restated in the following manner:

$U(\Pi_H) = Q P_0^T - a Q^2\sigma^2_p;$

Let us observe that during a corner the futures price is disconnected from the spot price. Given that the storage capacity is saturated at the delivery point, producers cannot sell their commodity on the spot market at the price *D*. This is because the potential buyers are not able to deliver the commodity against the futures contracts. Thus, the spot price remains unaffected by the rise of the futures price. In period 0 the expected utility of an unhedged producer remains the same and is independent of the manipulator's strategies.

In the case of manipulation the expected utility of a hedged producer depends on the probability of being able to deliver the commodity to the manipulator:

$$U(\Pi_H) = \left(1 - \frac{S}{X}\right)\left(QP_1^L - QD\right) + QP_0^T - a\left[\left(\frac{S}{X} - \left(\frac{S}{X}\right)^2\right)\left(QP_1^L - QD\right)^2\right]$$

which is less than her expected utility when there is no manipulation (QP_0^T).

Furthermore we assume that the hedger's expected utility is lower in the case of a corner (when she becomes a victim of the manipulator) than in the situation when she stays out of the futures market.

Given these assumptions we present the possible outcomes of the different strategies in the decision matrix displayed in Table 1:

Table 1. ***Manipulator's and hedger's decision matrix***

	M	NM
NH	$S P_1^L - S P_0^F$; $Q P_0^T - a Q^2\sigma^2_p$	0 ; $Q P_0^T - a Q^2\sigma^2_p$
H	$(X - S)(D-P_0^F) + S P_1^L - S P_0^F$ $(1-S/X)(QP_1^L - QP_0^F) + a[(S/X-(S/X)^2]\,(QP^L_1-QD)^2$	0 ; $Q P_0^F$

We observe that there is no pure strategy Nash-equilibrium in that game. If the manipulator chooses to initiate a corner the best hedger's strategy is to stay out of the market. In that case the manipulator makes a loss and therefore prefers not to enter in the futures markets. In that situation the producer would be better off hedging, but that would enable the manipulator to launch a corner and make a positive profit.

Proposition 1. *Recognizing that the pure strategy Nash-equilibrium does not exist, the players will turn to mixed strategies, i.e. they will determine the probabilities of using each of the strategies available. We define q as the probability that a commodity producer will hedge with futures contracts and p as the probability that a manipulator will initiate a corner. In equilibrium*:

$$q = \frac{S\left(P_0^T - P_1^L\right)}{\left(X - S\right)\left(D - P_0^T\right)}$$

and

$$p = \frac{aQ^2 o_p^2}{\left(1 - \frac{S}{X}\right)\left(QD - QP_1^L\right) + a\left(\frac{S}{X} - \left(\frac{S}{X}\right)^2\right)\left(QP_1^L - QD\right)^2}$$

Proof: See Appendix 1.

The analysis of squeezes in probabilistic terms sets a convenient framework for determination of factors influencing the frequency of futures markets manipulation. We emphasize the role of penalty fines and storage capacity behind these attempts.

Result 1. *Intensity of hedging is directly proportional to storage space available and inversely proportional to amount of penalty fines, while probability of manipulation increases with this amount. The probability of manipulation increases also with spot price variability and and hedgers' risk aversion:*

$$\frac{\delta q}{\delta D} < 0$$

$$\frac{\delta q}{\delta S} > 0$$

and

$$\frac{\delta p}{\delta a} > 0$$

$$\frac{\delta p}{\delta o_p^2} > 0$$

$$\frac{\delta p}{\delta D} > 0$$

Proof: See Appendix 1.

The probability of hedging depends on two parameters, the default penalty paid by the shorts unable to deliver *D* and the storage capacity at the point of delivery *S*.

The higher the default penalty, the lower is the frequency of use of the futures markets by the commodity producers. In fact, the higher the level of *D*, the more profitable is the corner. That would encourage the manipulator to initiate corners more frequently, making the futures markets less attractive to the hedgers. Therefore the futures exchanges' governing bodies should set the default penalty at a reasonable level in order to promote hedging and discourage potential manipulators.

The interpretation of the influence of the storage capacity *S* on the hedging activity and on the probability of corners is quiet straightforward. The expansion of this capacity makes corners costlier and thus less frequent, which encourages producers to use futures markets for hedging purposes.

We also observe that the probability of manipulation depends on the coefficient of risk aversion of the hedgers. In fact the more risk averse they are, the more likely they are to enter the futures market, providing the manipulator with the needed "cover" to successfully launch the corner.

CONCLUSION

According to historical examples, confirmed by theoretical modeling, the storage capacity and the level of default penalty imposed on the shorts unable to deliver the commodity against futures contracts played the crucial role in the development of corners in the nineteenth century.

The expansion of storage capacity reduces the probability of manipulation *ex ante* but it could also be a useful tool in breaking up an actual corner as an *ex post* measure. The exchange authorities have the ability to declare more elevators regular thus making the corner harder to accomplish. That was the case during Lyon's attempted corner in 1872. At present the exchanges deter corners by allowing for delivery of a range of different qualities of a given commodity in a number of places.

The problem of a default penalty imposed on the shorts unable to deliver the physical commodity against the futures contracts is more delicate. That penalty was set up in order to compel the traders to respect their commitments and thus enhance the efficiency of futures markets. It seems paradoxical that this penalty makes the corners more profitable, thus encouraging manipulation and reducing the usefulness of the futures markets for hedgers and speculators - their legitimate users.

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END NOTES

1. Pirrong (1995b) records 124 cases of attempted manipulation on the CBOT between 1868 and 1924.
2. this unit of measure is equivalent to 8 gallons or roughly 25 kg.
3. "Regular" means officially recognized by the CBOT as an elevator where the deliveries against futures contracts can be made.
4. As was his aura. At that time the traders sang

   I see Old Hutch start for the club
   Goodbye my money, goodbye,
   He’s given us a pretty rough rub,
   Goodbye my money, goodby.
5. The superscripts *C* and *F* mean stand for Cash and Futures.

APPENDIX 1

In a game played with mixed strategies each player plays any given strategy with a certain probability. She maximizes her expected profit in equilibrium assuming that the probabilities chosen by her adversary are held constant. From the confrontation of these two maximization programs arises a Nash mixed strategy equilibrium.

Let us assume that the manipulator plays the strategy "to Manipulate" (M) with probability *p* and the strategy "Not to Manipulate" (NM) with probability *1-p* and that the hedger chooses the strategy "to Hedge" (H) with the probability *q* and the strategy "Not to Hedge" with probability *1-q*. We obtain the equilibrium probabilities by resolving the following programs:

$$\underset{p}{Max}\, E(\Pi_M)$$

and

$$\underset{q}{Max}\, E(\Pi_H)$$

The manipulator's expected profit equals:

$E(\Pi_M) = pq \times [\ (X - S)\ (D-P_0^F) + SP_1^L - S\ P_0^F\ ] + p(1-q) \times [\ SP_1^L - S\ P_0^F]$
$+ (1-p)(1-q) \times 0 + (1-p)q \times 0.$

The first order condition to maximize her profit solves the following equation:

$$\frac{\delta E(\Pi_M)}{\delta p} = 0$$

Then the following condition must be satisfied:

$$\frac{\delta E(\Pi_M)}{\delta p} = \left[SP_1^L - SP_0^F\right] + q \times \left[(X-S)(D-P_0^F\right] = 0$$

yielding:

$$q = \frac{S\left(P_0^T - P_1^L\right)}{\left(X-S\right)\left(D-P_0^T\right)}\,.$$

We can easily check that this term decreases with *D* and increases with *S*. Thus:

$$\frac{\delta q}{\delta D} < 0$$

and

$$\frac{\delta q}{\delta S} > 0$$

In the hedger's case, his expected profit is expressed the following equation:

$$E(\Pi_H) = p(1-q) \times [QP_0^F - aQ^2\sigma_p^2] + (1-p)(1-q) \times [QP_0^F - aQ^2\sigma_p^2]$$
$$+ pq \times [(1-S/X)(QP_1^L - QD) + QP_0^F - a(S/X - (S/X)^2)(QP_1^L - QD)^2] + (1-p)q \times QP_0^F$$
$$= QP_0^F - aQ^2\sigma_p^2 + q \times [aQ^2\sigma_p^2] + pq \times [(1-S/X)(QP_1^L - QD)$$
$$- a(S/X - (S/X)^2)(QP_1^L - QD)^2].$$

The solution of the next equation satisfies the first order condition to maximize hedger's profit:

$$\frac{\delta E(\Pi_H)}{\delta q} = aQ^2\sigma_p^2 + p \times \left[\left(1 - \frac{S}{X}\right)\left(QP_1^L - QD\right) - a\left(\frac{S}{X} - \left(\frac{S}{X}\right)^2\right)\left(QP_1^L - QD\right)^2\right] = 0$$

which yields:

$$p = \frac{aQ^2 o_p^2}{\left(1 - \frac{S}{X}\right)\left(QD - QP_1^L\right) + a\left(\frac{S}{X} - \left(\frac{S}{X}\right)^2\right)\left(QP_1^L - QD\right)^2}$$

It is straightforward to show that:

$$\frac{\delta p}{\delta a} > 0$$

$$\frac{\delta p}{\delta o_p^2} > 0$$

The probability of manipulation also increases with the amount of the penalty fee:

$$\frac{\delta p}{\delta D}=\frac{\left(a\sigma_p^2\right)\times\left[\left(1-\frac{S}{X}\right)Q+a\left(\frac{S}{X}-\left(\frac{S}{X}\right)^2\right)\left(-2Q^2P_1^L+2Q^2D\right)\right]}{\left[\left(1-\frac{S}{X}\right)\left(QD-QP_1^L\right)+a\left(\frac{S}{X}-\left(\frac{S}{X}\right)^2\right)\left(QP_1^L-QD\right)^2\right]^2}$$

Given that $D > P_1^L$ the term stated above is positive, thus:

$$\frac{\delta p}{\delta D}>0\,.$$